QCD calculations of thermal photon and dilepton production

François Gelis
gelis@th.u-psud.fr

L. P. T. , Université Paris Sud
Orsay, France

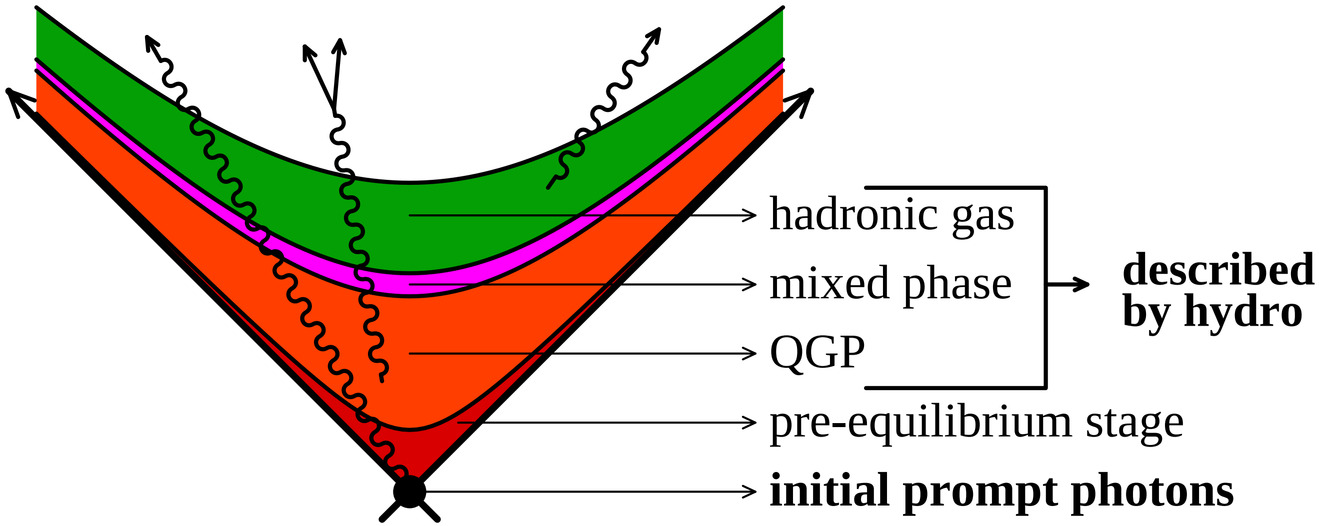
Outline

- What are electromagnetic probes good for ?
- Sketch of a heavy ion collision
- How to compute a thermal rate ?
- History
- Landau-Pomeranchuk-Migdal effect
- Dilepton production
- Quasiparticle models
- Lattice approach
- Non-equilibrium effects

What are E.M. probes good for?

- Photon rates are larger in hotter matter
  ▷ probe early stages of the system evolution
  ▷ very sensitive to the initial temperature
  ▷ How early ? Photons are produced after quarks have been produced
- Photons have a very large mean free path
  ▷ they escape without reinteractions
- Note: some other relevant talks:
  ▷ I. Sarcevic: γ and π^0 from initial partons
  ▷ F. Karsch: lattice calculations
  ▷ Panel discussion on photons:
  V. Ruuskanen, A. Dumitru, H. Zaraket, S. Rasanen
  ▷ K. Reygers, I. Johnson, R. Averbeck: experiments

Sketch of a heavy ion collision



▷ Separate description of prompt photons and thermal photons

▷ Usual assumption: very few quarks in the pre-equilibrium phase + short duration $\longrightarrow$ almost no photon production

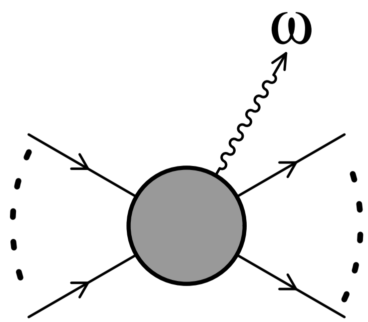
▷ One needs:

- ▷ prompt photon cross-section
- ▷ rates from QGP
- ▷ rates from hadronic gas

How to compute a rate ?

- Pedestrian approach:

$$\omega \frac{dN_\gamma}{dt dV d^3\vec{\boldsymbol{q}}} \propto \int_{\substack{\text{unobserved}\\ \text{particles}}} \left| \quad \right|^2$$



$$\times\, n(\omega_1)\cdots n(\omega_n)$$

$$\times\, (1 \pm n(\omega'_1))\cdots(1 \pm n(\omega'_p))$$

- Using Thermal Field Theory:

  Weldon (1983) - Gale, Kapusta (1991)

$$\omega \frac{dN_\gamma}{dt dV d^3\vec{\boldsymbol{q}}} \propto \frac{1}{e^{\omega/T} - 1}\, \mathrm{Im}\, {\Pi_{\mathrm{ret}}}^{\mu}{}_{\mu}(\omega, \vec{\boldsymbol{q}})$$

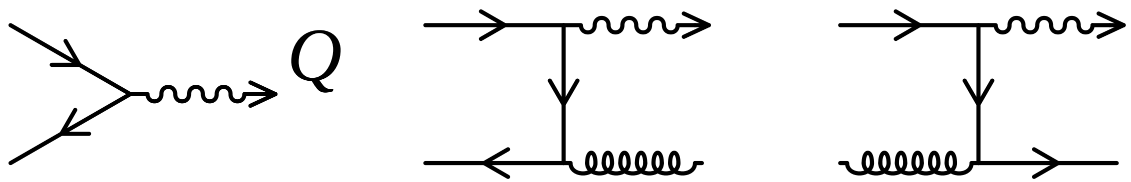
History (1/3)

- Pre-HTL calculations

  McLerran, Toimela (1985) - Baier, Pire, Schiff (1988)

  Altherr, Aurenche, Becherrawy (1989)

  Altherr, Ruuskanen (1992)

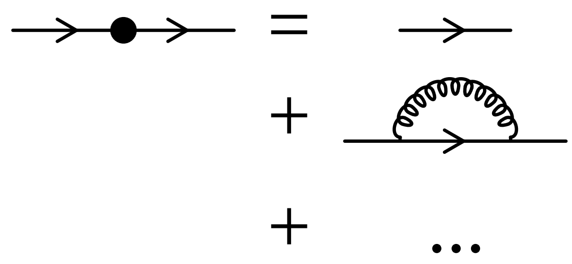


$\triangleright$ For real photons ($Q^2 = 0$), infrared divergence when the quark exchanged in the t-channel is massless:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}(\omega, \vec{\boldsymbol{q}}) \propto \alpha\alpha_s \ln(\omega T/Q^2)$$

History (2/3)

- The Hard Thermal Loop revolution (1990)
  Braaten, Pisarski (1990) - Frenkel, Taylor (1990)



Medium effects give a mass to quasi-particles:
$m_{\rm th} \sim \sqrt{\alpha_s} T$

$\triangleright$ $m_{\rm th}$ screens the infrared divergence:

Kapusta, Lichard, Seibert (1991)

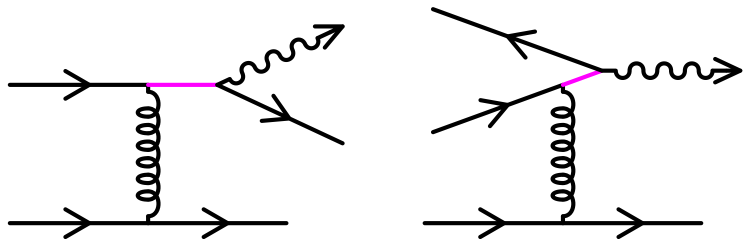
Baier, Nakkagawa, Niegawa, Redlich (1992)

$$\mathrm{Im}\,\Pi_{\rm ret}{}^{\mu}{}_{\mu}(\omega,\vec{\boldsymbol{q}}) = 4\pi\alpha\alpha_s T^2 \left[\ln\left(\frac{\omega T}{m_{\rm th}^2}\right) - 0.027\right]$$

History (3/3)

- Collinear enhancement

Aurenche, F.G., Kobes, Petitgirard (1996)
Aurenche, F.G., Kobes, Zaraket (1998)



singularity when a real photon is emitted forward: $\alpha_s^2 \frac{T^2}{m_{\rm th}^2} \sim \alpha_s$

Mohanty (2000) - Steffen, Thoma (2001)
Aurenche, F.G., Zaraket (2002) For 3 colors and 2 flavors:

$$\mathrm{Im}\, \Pi_{\rm ret}{}^{\mu}{}_{\mu}(\omega, \vec{\boldsymbol{q}}) = \frac{32}{3\pi} \alpha \alpha_s \left[\pi^2 \frac{T^3}{\omega} + \omega T \right]$$

Notes

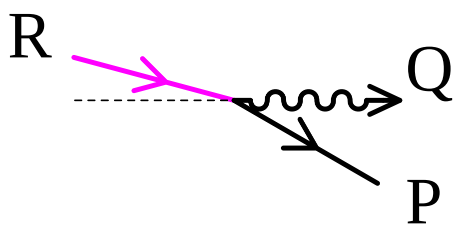
- For 3 colors and 3 flavors:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(\omega,\vec{\boldsymbol{q}}) = \frac{32}{3\pi}\left[1+\frac{5\pi^2}{36}+\ln\left(\frac{\sqrt{2}}{3}\right)-\frac{55}{12}\ln^2(2)\right.$$
$$\left.+\frac{10}{3}\ln(2)\ln(3)-\frac{5}{3}\mathrm{Li}_2\left(\frac{3}{4}\right)-\frac{5}{3}\mathrm{Li}_2\left(-\frac{1}{2}\right)\right]$$
$$\times\alpha\alpha_s\left[\pi^2\frac{T^3}{\omega}+\omega T\right]$$

- More generally, the prefactor is a function of $m_{\mathrm{th}}/m_{\mathrm{debye}}$. ▷ In the HTL framework, this ratio is independent on T.

LPM effect (1/6)

- Photon formation time



$$R \equiv P + Q$$
$$(r_0 = p_0 + \omega)$$
$$P^2 = m_{\rm th}^2$$

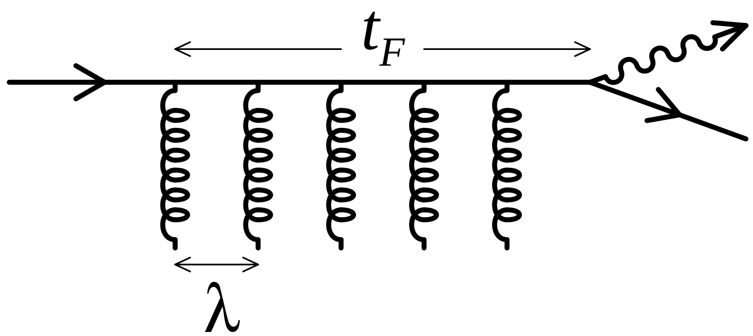
$$t_{_F}^{-1} \sim \delta E = r_0 - \sqrt{\vec{\boldsymbol{r}}^2 + m_{\rm th}^2}$$

$$\boxed{t_{_F}^{-1} = \frac{\omega}{2p_0r_0}\left[\vec{\boldsymbol{p}}_\perp^2 + m_{\rm th}^2\right]}$$

LPM effect (2/6)

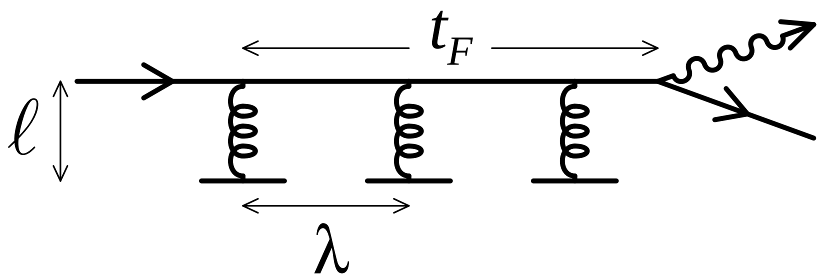
- Other scales
  ▷ Mean free path: $\lambda \sim (\alpha_s T \ln(1/\alpha_s))^{-1}$
  ▷ Electric screening: $\ell_{\rm elec} \sim (\sqrt{\alpha_s} T)^{-1}$
  ▷ Magnetic screening: $\ell_{\rm mag} \sim (\alpha_s T)^{-1}$
- LPM effect: multiple scatterings are important if

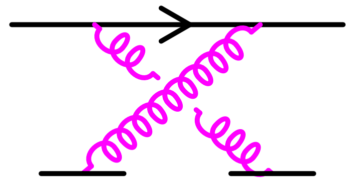
$$\boxed{t_F \gtrsim \lambda}$$



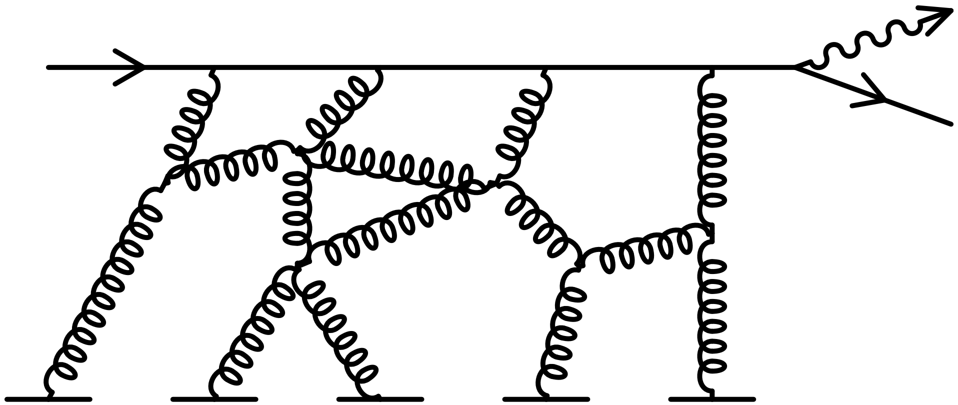
LPM effect (3/6)

- How bad could it be?
  - Short ranged interactions: $\ell \ll \lambda$





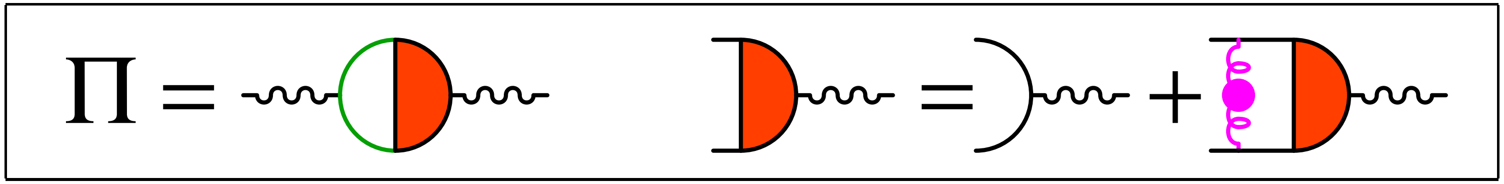
  - Long ranged interactions: $\ell \gtrsim \lambda$



LPM effect (4/6)

Arnold, Moore, Yaffe (2001,2002)

- A cancellation between vertex and self-energy topologies prevents long ranged interactions
  ▷ Only ladder topologies contribute to $\Pi_{\rm ret}$:



$$\mathrm{Im}\,\Pi_{\rm ret}{}_\mu^\mu = \alpha\ \mathrm{Im}\int dp_0\, d^2\vec{\boldsymbol{p}}_\perp\,[\cdots]\ 2\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp)$$

$$\frac{\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp)}{t_F} = 2\vec{\boldsymbol{p}}_\perp \oplus i\alpha_s T\int d^2\vec{\boldsymbol{l}}_\perp\,\mathcal{C}(\vec{\boldsymbol{l}}_\perp)[\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp) - \vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{l}}_\perp)]$$

LPM effect (5/6)

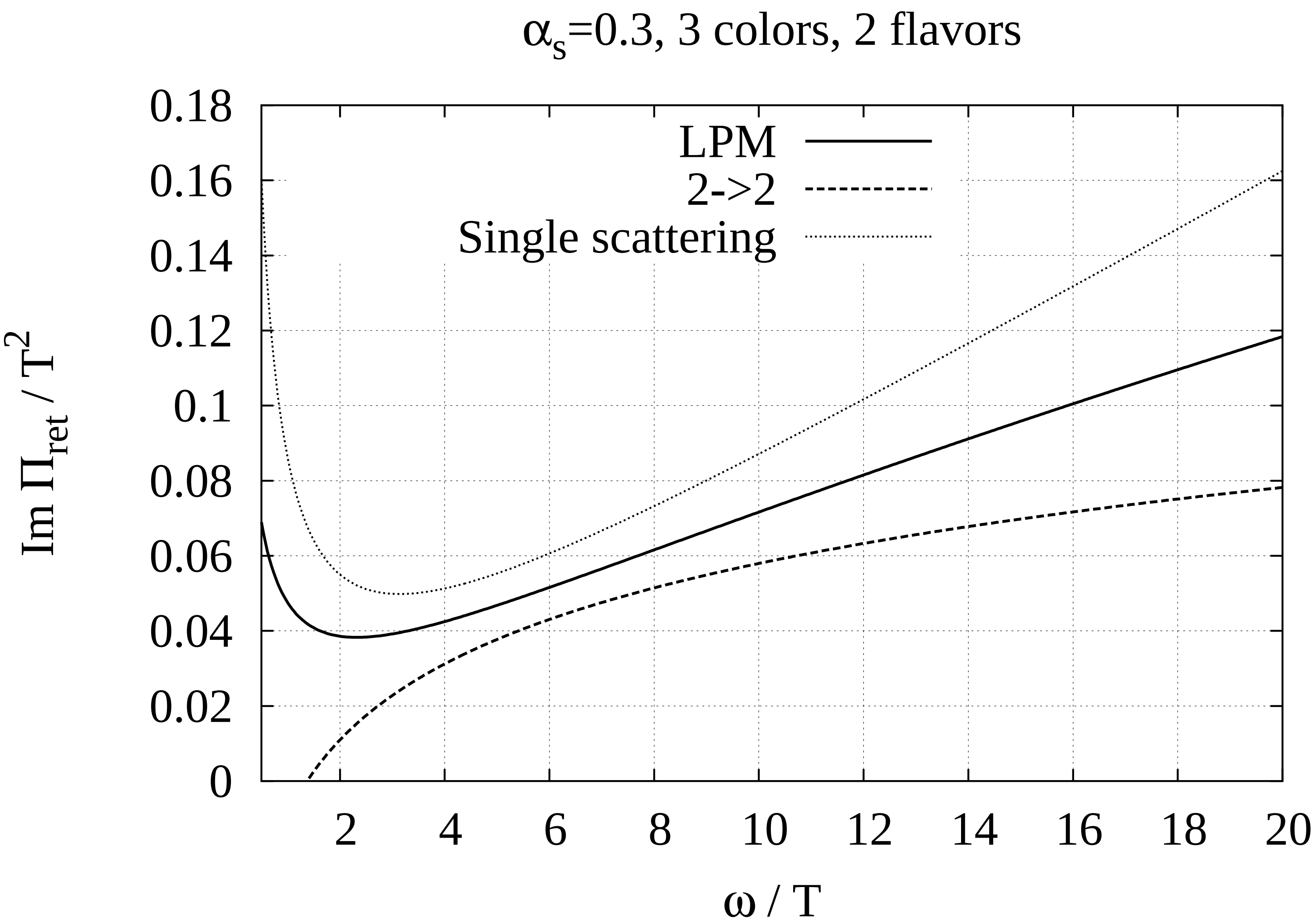
- The collision integral is given by:

$$\mathcal{C}(\vec{\boldsymbol{l}}_\perp) = \frac{1}{\vec{\boldsymbol{l}}_\perp^{\,2}} - \frac{1}{\vec{\boldsymbol{l}}_\perp^{\,2} + m_{\rm debye}^2}$$

- Notes:
  $\triangleright$ all ladder diagrams contribute to $\mathcal{O}(\alpha\alpha_s)$:
  each extra gluon brings $\alpha_s T p_0 r_0 / \omega m_{\rm th}^2$
  $\triangleright$ the LPM-resummed result depends only on
  $m_{\rm th}$ and $m_{\rm debye}$

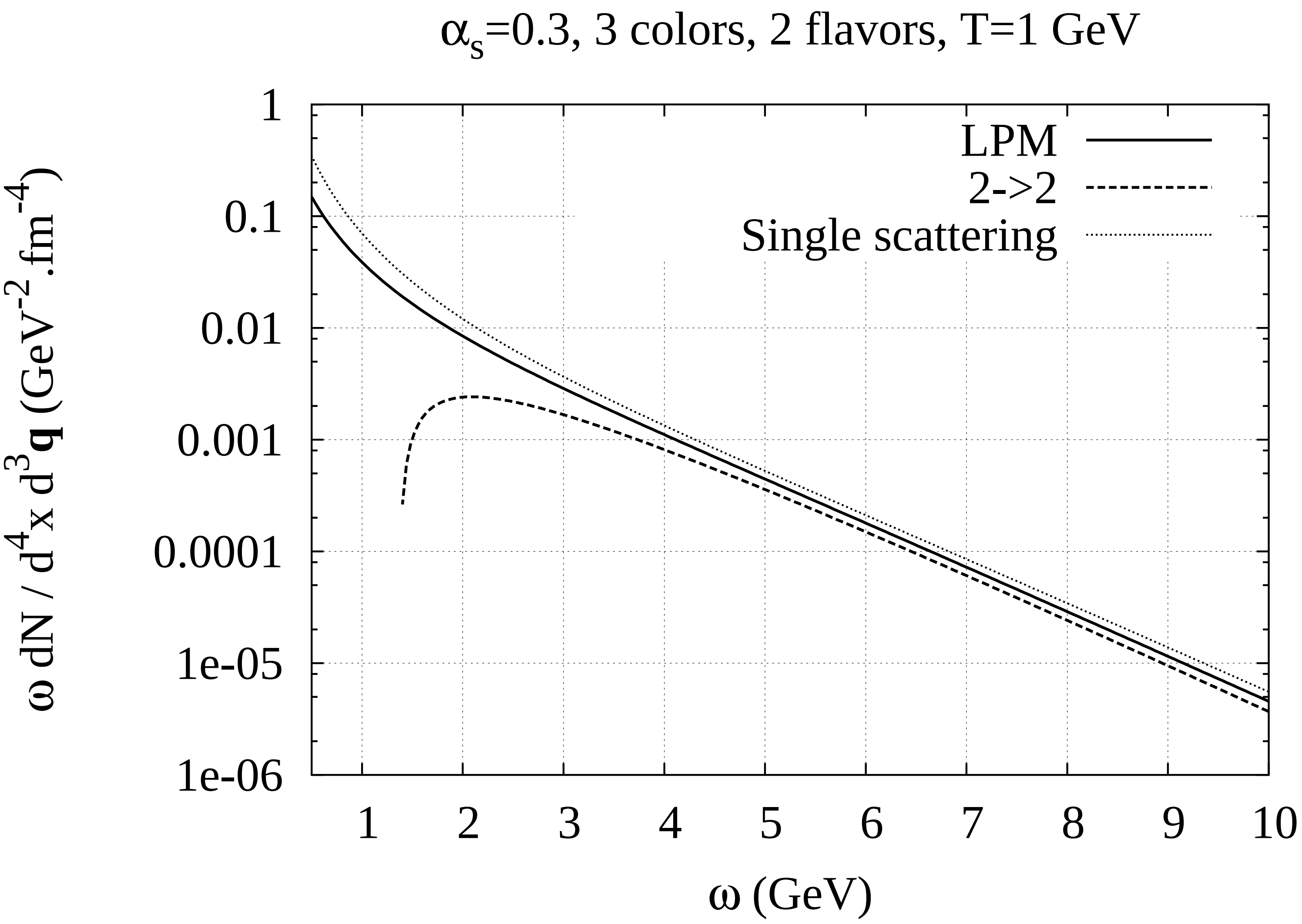
LPM effect

- Summary: photon production at $\mathcal{O}(\alpha\alpha_s)$



LPM effect (6/6)

- Physical photon rate at $\mathcal{O}(\alpha\alpha_s)$



Dilepton production (1/2)

- What's different?

  ▷ $$\frac{dN_{l^+l^-}}{dtdV\,d^4Q} \propto \frac{\alpha}{Q^2}\frac{1}{e^{\omega/T}-1}\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(Q)$$

  ▷ For $Q^2 \geq 4m_{\mathrm{th}}^2$, there is a contribution at $\mathcal{O}(\alpha^2)$ from the process $q\bar{q} \to \gamma^* \to l^+l^-$

  ▷ The longitudinal mode of the γ^* contributes

  Aurenche, F.G., Moore, Zaraket (preliminary)

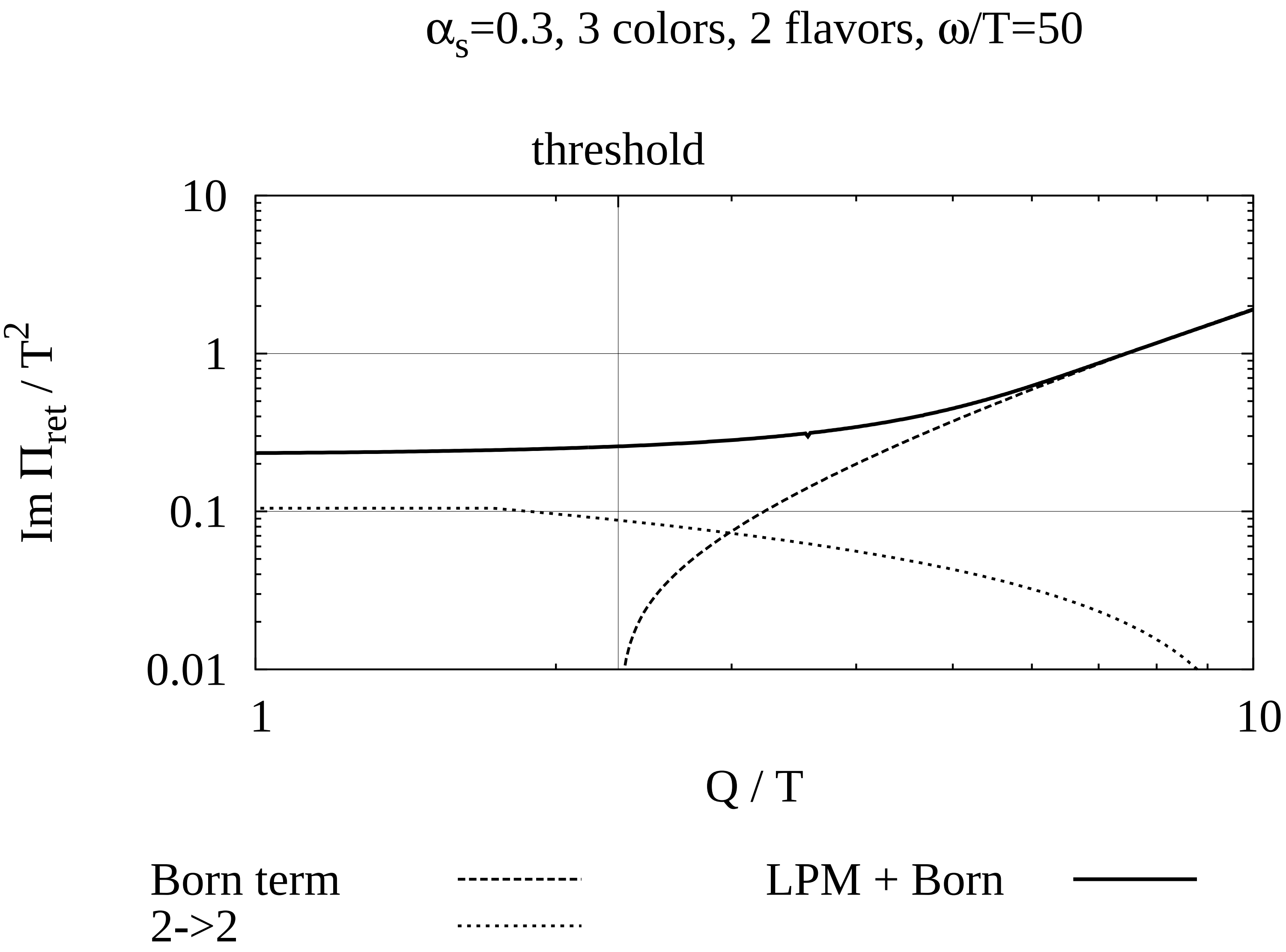
$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}_{\mu} = \alpha\;\mathrm{Im}\int dp_0\, d^2\boldsymbol{\vec{p}}_\perp\,[\cdots]\,\left[2\boldsymbol{\vec{p}}_\perp\cdot\boldsymbol{\vec{f}}(\boldsymbol{\vec{p}}_\perp)\oplus Q^2 g(\boldsymbol{\vec{p}}_\perp)\right]$$

$$\frac{g(\boldsymbol{\vec{p}}_\perp)}{t_{_F}} = 1\oplus i\alpha_s T\int d^2\boldsymbol{\vec{l}}_\perp\,\mathcal{C}(\boldsymbol{\vec{l}}_\perp)[g(\boldsymbol{\vec{p}}_\perp)-g(\boldsymbol{\vec{p}}_\perp+\boldsymbol{\vec{l}}_\perp)]$$

▷ Note: these equations contain the Born term ($q\bar{q}\to\gamma^*$)

Dilepton production

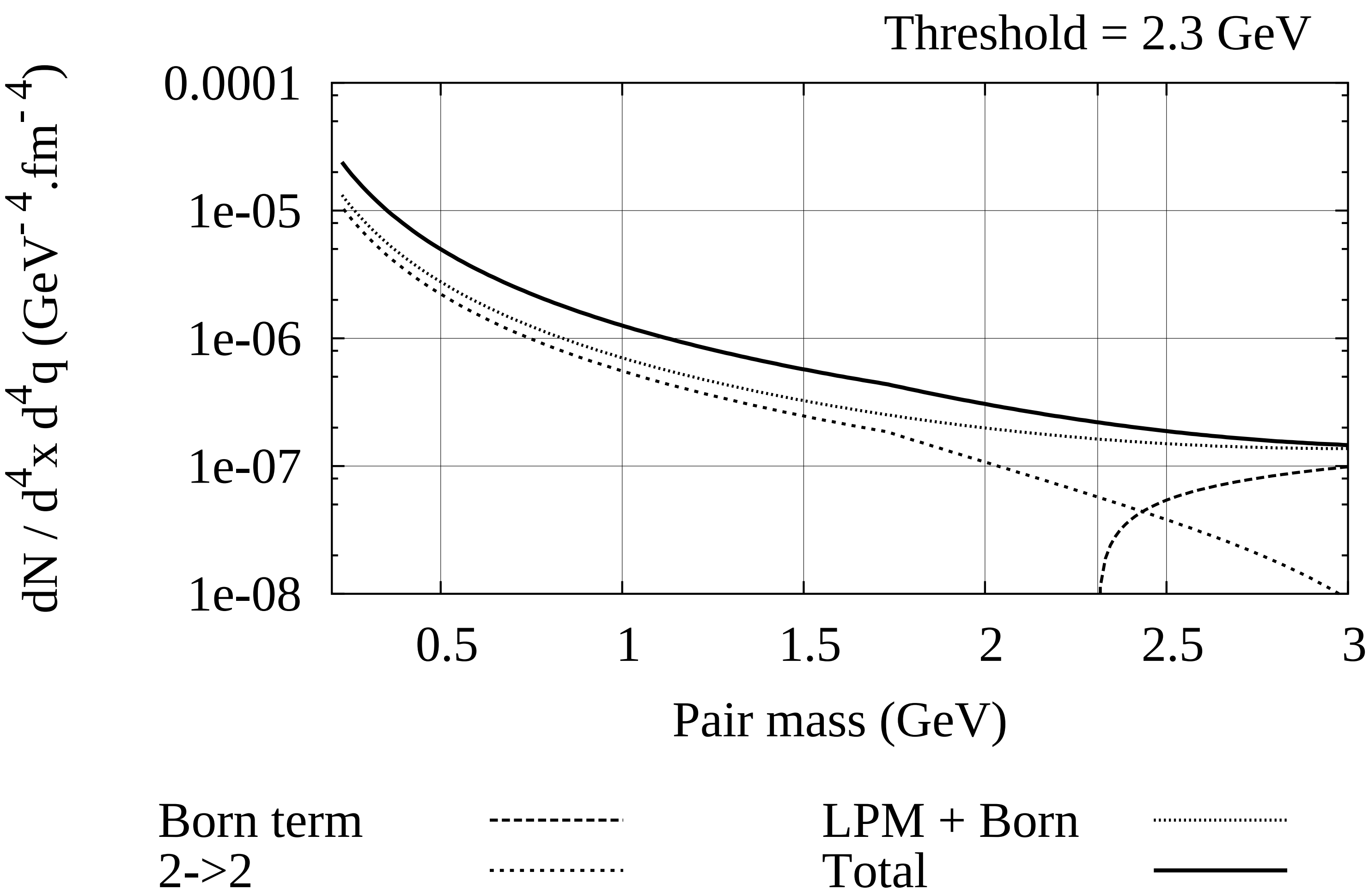
- Summary: all terms up to $\mathcal{O}(\alpha^2\alpha_s)$ (preliminary)



Dilepton production (2/2)

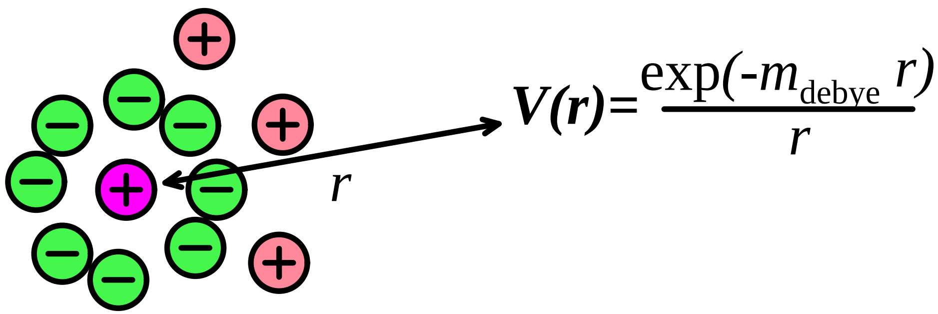
- Physical dilepton rate at $\mathcal{O}(\alpha^2\alpha_s)$ (preliminary)

α_s=0.3, 3 colors, 2 flavors, T=1GeV, ω=5GeV



Quasiparticle models (1/2)

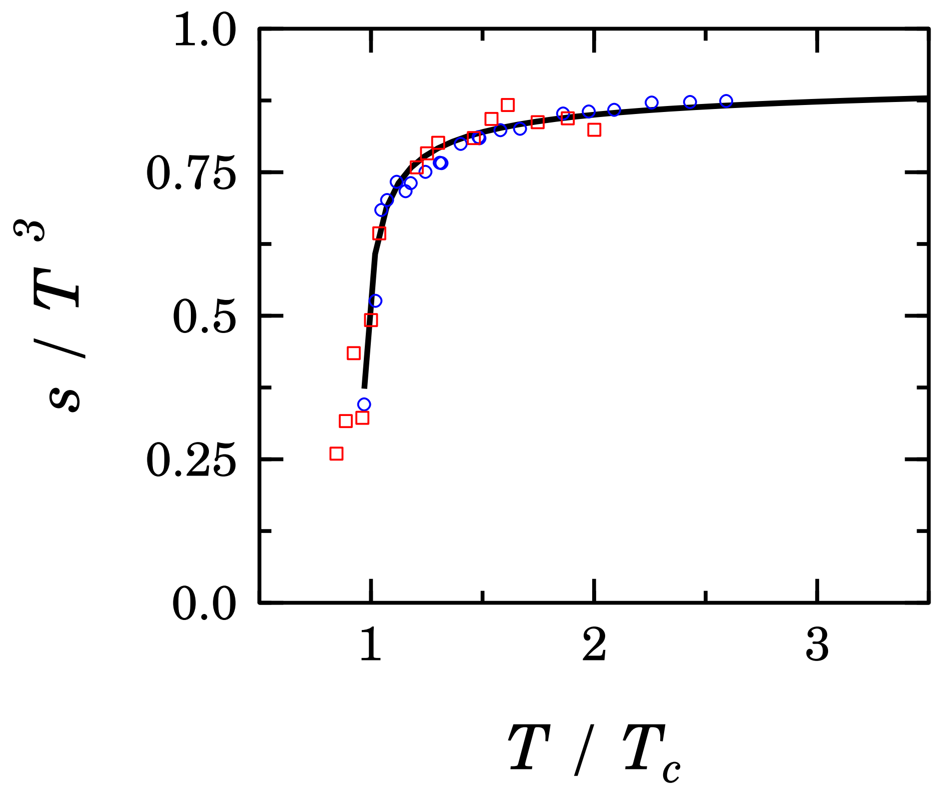
- Photon/dilepton rates depend on $m_{\rm th}/m_{\rm debye}$
- HTL prediction: $m_{\rm th}/m_{\rm debye}$ is a constant
- On physical grounds, this cannot be exact:

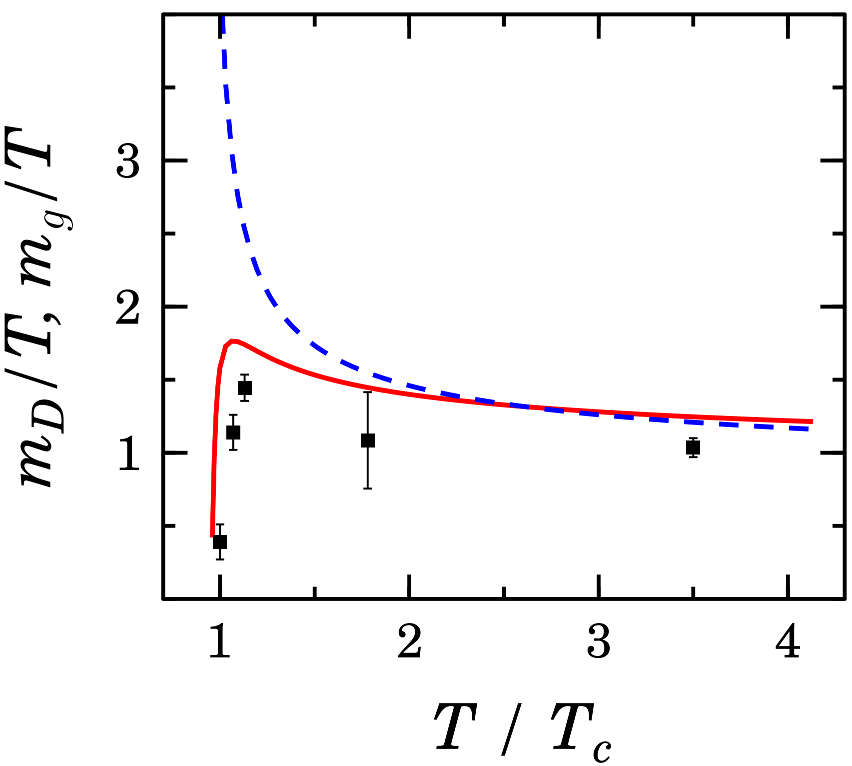


Heavy quasiparticles (large $m_{\rm th}$)
$\triangleright$ medium difficult to polarize
$\triangleright$ small $m_{\rm debye}$

Quasiparticle models (2/2)

- Quasiparticle fits of the lattice entropy (or pressure) require a large m_{th} near T_c
  $\triangleright$ m_{debye} computed in this model is small near T_c





Peshier, Kampfer, Soff (1996,2001,2002)

Lattice approach: method (1/4)

Karsch, Laermann, Petreczky, Stickan, Wetzorke (2001,2002)

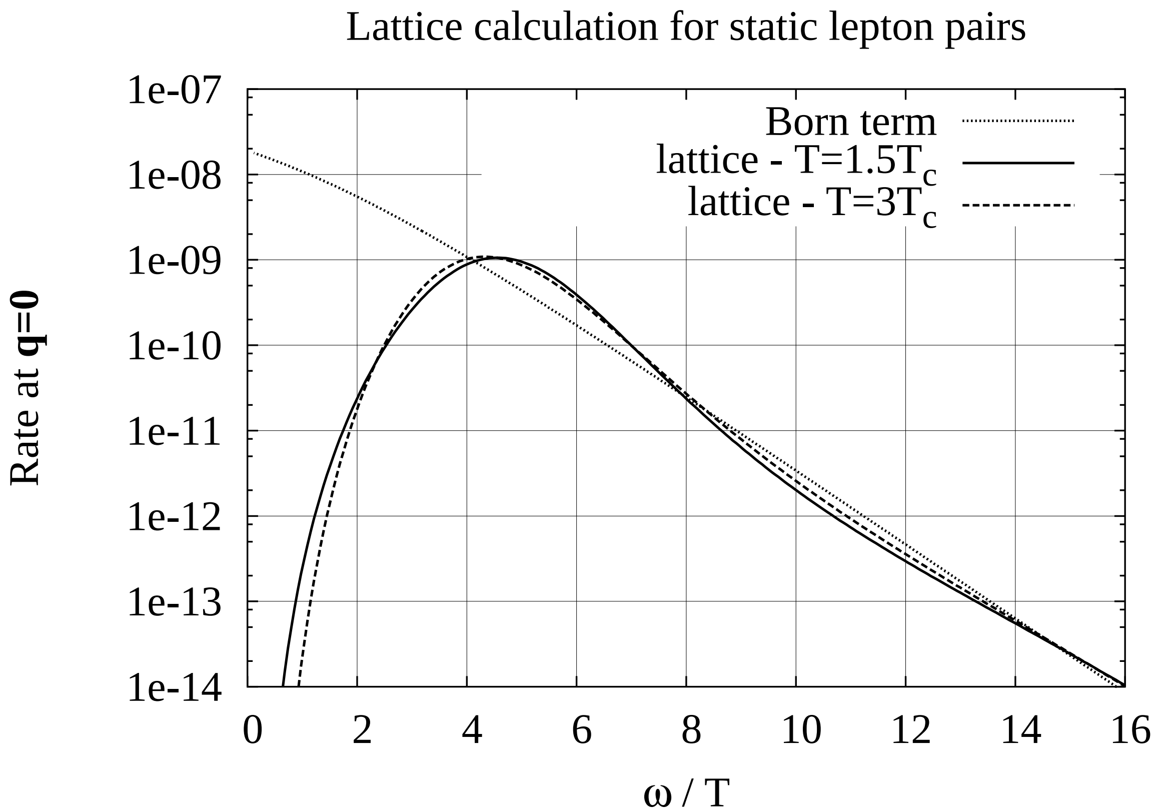
- How to calculate photon rates on the lattice?
  ▷ Measure the Euclidean vector-vector correlator $\Pi(\tau, \vec{\boldsymbol{q}})$ on the lattice
  ▷ Invert the following dispersion relation:

$$\Pi(\tau, \vec{\boldsymbol{q}}) = \int_0^\infty d\omega \, \mathrm{Im}\, \Pi(\omega, \vec{\boldsymbol{q}}) \, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

($\tau \in [0, 1/T]$ is the Euclidean time)

- Main difficulty:
  ▷ $\Pi(\tau, \vec{\boldsymbol{q}})$ is known at discrete points
  ▷ Maximum Entropy Method (F. Karsch's talk): extra constraint on $\mathrm{Im}\, \Pi$

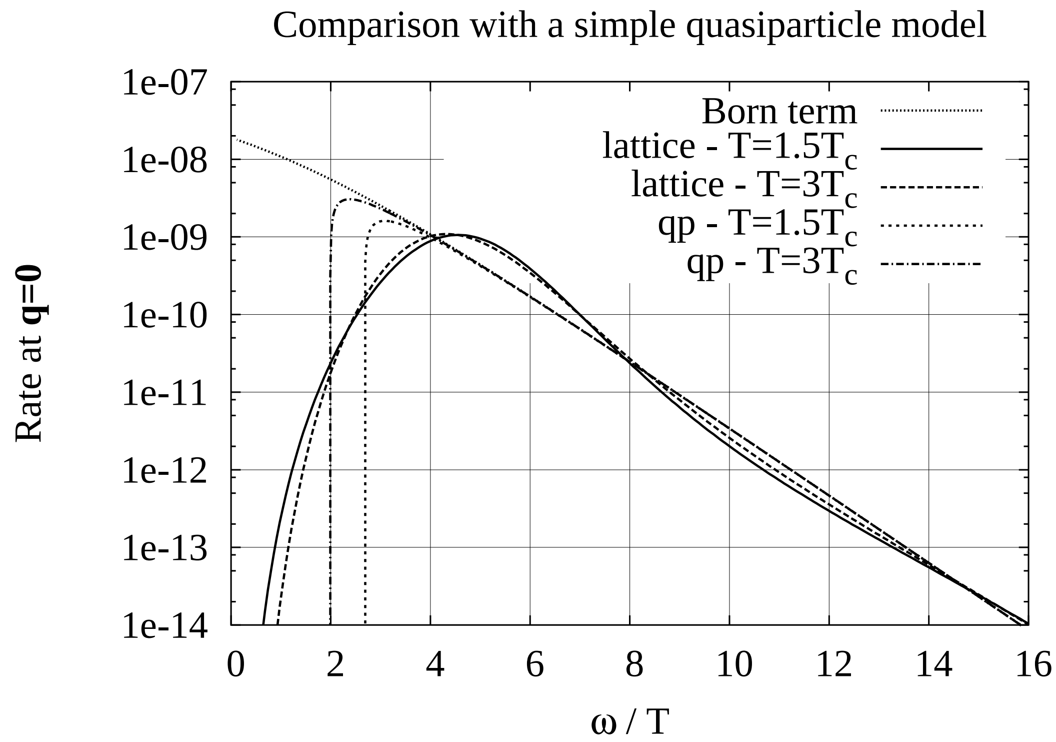
Lattice approach: result (2/4)



- Most striking features:
  - ▷ rather close to the Born term
  - ▷ fast drop of the rate for $\omega \lesssim 3T$
  - ▷ scales like a universal function of ω/T

Lattice approach: remarks (3/4)

- $q\bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$ with massive quarks



- Quark thermal mass: $m_{\text{th}}^2 \propto \alpha_s(T)T^2$
  $\triangleright$ The running of $\alpha_s(T)$ induces scaling violations

Lattice approach: remarks (4/4)

- Setting $\tau = 1/2T$, we have

$$\int_0^\infty d\omega \frac{\mathrm{Im}\,\Pi(\omega,\vec{\boldsymbol{q}})}{\sinh(\omega/2T)} = \Pi(1/2T,\vec{\boldsymbol{q}}) < \infty$$

$\triangleright$ perturbative results break down at small ω

- From the electric conductivity

$$\sigma_{\mathrm{el}} = \frac{1}{6}\lim_{\omega\to 0}\frac{\mathrm{Im}\,\Pi(\omega,\vec{0})}{\omega}$$

$\triangleright$ $\mathrm{Im}\,\Pi(\omega,\vec{0}) \propto \omega$ when $\omega \longrightarrow 0$

$\triangleright$ $\sigma_{\mathrm{el}} \neq 0$ in a QGP $\triangleright$ $\mathrm{Im}\,\Pi(\omega,\vec{0})/\omega^2 \longrightarrow \infty$

$\triangleright$ the lattice calculation breaks down at small ω

Non-equilibrium effects (1/3)

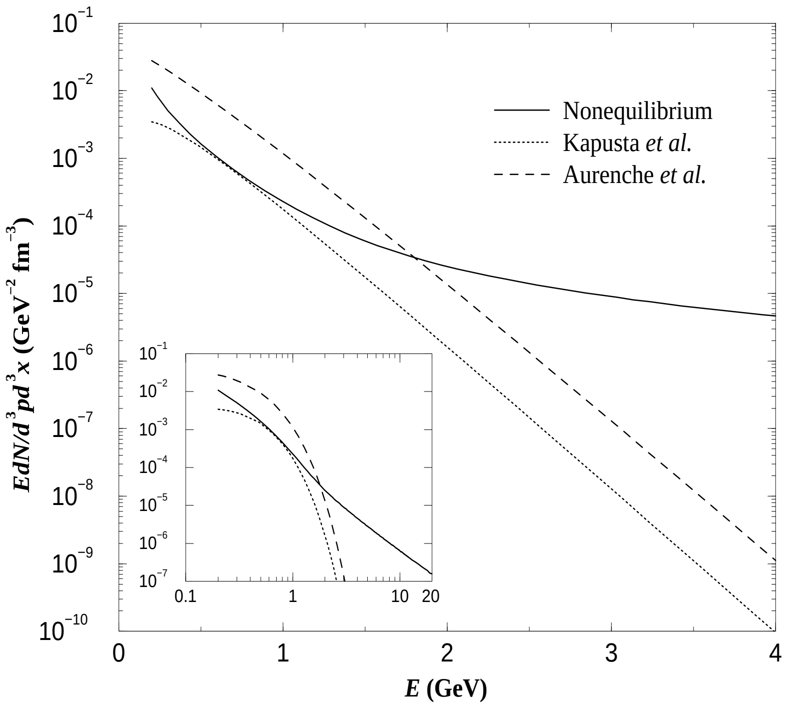
- Can we use the equilibrium rates in hydro ?
  $\triangleright$ non equilibrium is not a problem if the distributions $n(\omega)$ are constant over scales $\gtrsim t_{_F}$
  $\triangleright$ $g = 2$, $T = 0.5\,\text{GeV}$, $\omega = 2\,\text{GeV}$: $t_{_F} \sim 0.3\,\text{fm}$

  $\triangleright$ Caveat: if there is no chemical equilibrium, one cannot use TFT simplifications

  $\triangleright$ simple model: $n(\omega) = \lambda/(\exp(\omega/T) \pm 1)$
  $(\lambda \neq 1\ , \quad \lambda_{\text{quark}} \neq \lambda_{\text{gluon}})$
  Traxler, Thoma (1996)
  Baier, Dirks, Redlich, Schiff (1997)
  Dutta, Sastry, Mohanty, Kumar, Choudhury (2001)

Non-equilibrium effects (2/3)

- Transient effects ?
  Wang, Boyanovsky (2000)



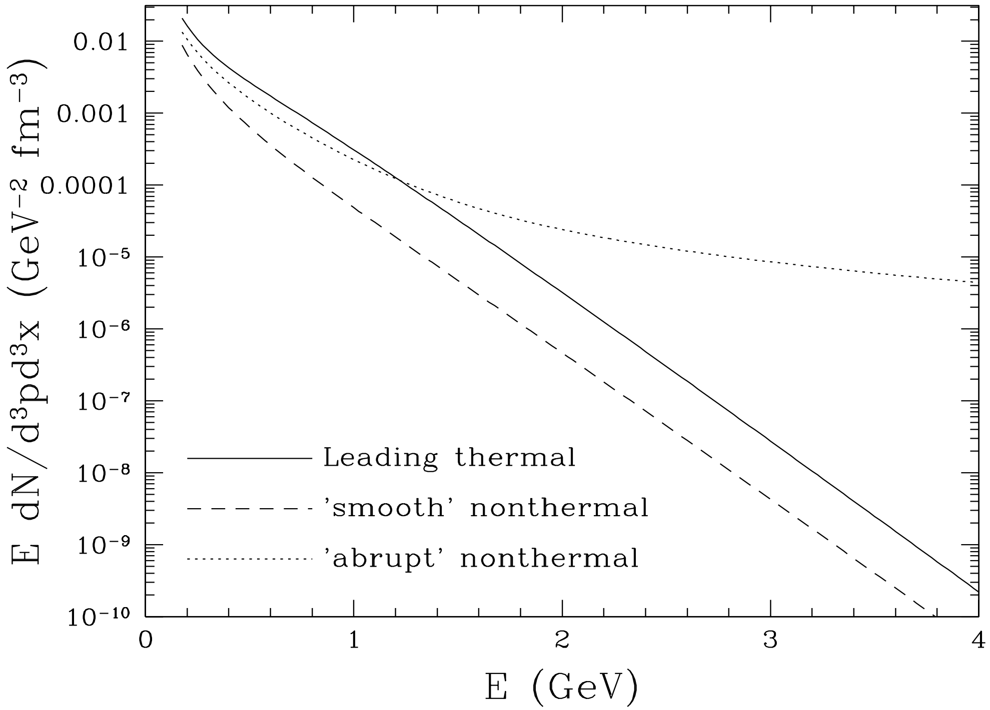
$g = 2, T = 200\,\mathrm{MeV}$
$t_f - t_i = 10\,\mathrm{fm}$

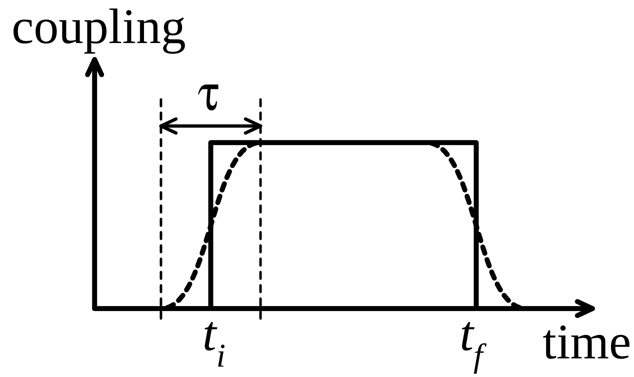
- $q\bar{q} \to \gamma$ is forbidden by E conservation
- does not hold if the system exists only during some finite window in time
- Model: switch on the interations in $[t_i, t_f]$

  $\triangleright$ power law spectrum (instead of $\exp(-\omega/T)$)

Non-equilibrium effects (3/3)

- What's wrong with this?
  ▷ total radiated energy is infinite !!
  ▷ This is due to the 'abrupt' switching of the interactions Moore (private communication)





$t_f - t_i = 10\,\text{fm} \quad \tau = 1\,\text{fm}$

▷ the asymptotic behavior becomes $\exp(-\tau\omega)$

▷ with $\tau = 1$ fm, this contribution is negligible

Conclusions (1/2)

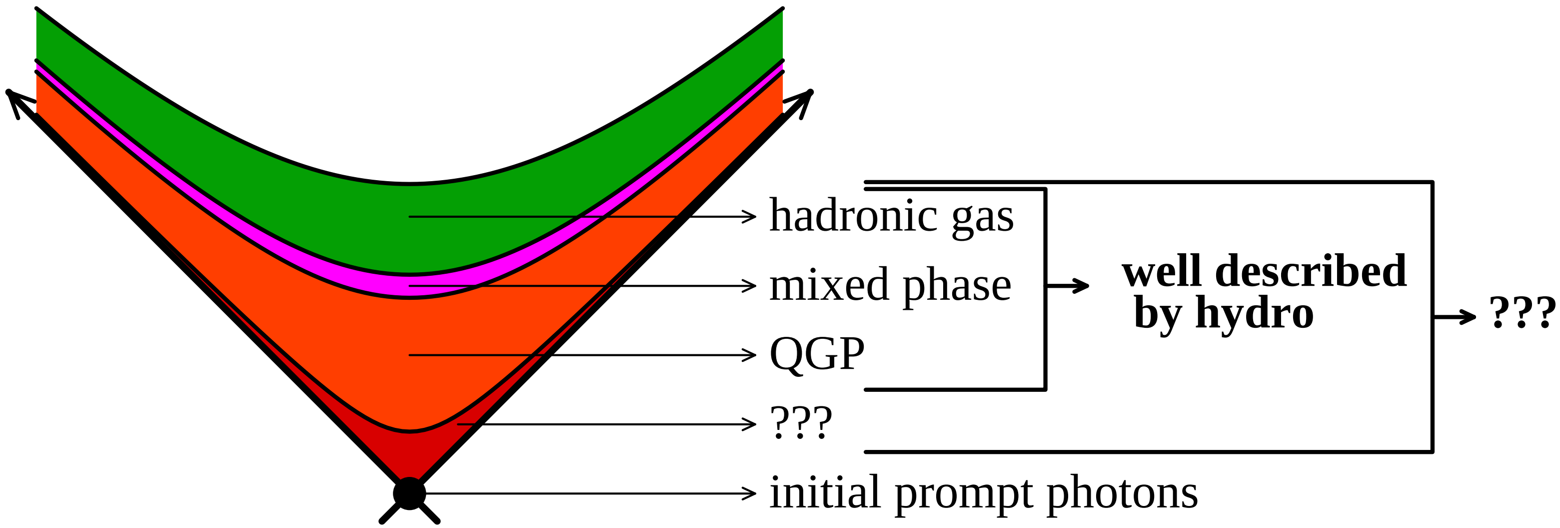
- Mostly under control:
  $\triangleright$ all thermal rates are known at $\mathcal{O}(\alpha_s)$, and the LPM suppression is rather small in the relevant energy range
  $\triangleright$ Lattice calculations of dilepton rates are now doable in principle

- Necessary improvements:
  $\triangleright$ better description of the quasiparticles
  $\triangleright$ extension to chemical non-equilibrium
  $\triangleright$ extend the lattice calculation to real photons

Conclusions (2/2)

- Puzzling issue:

  ▷ Can we go smoothly from initial prompt photon production to later thermal production ?



  ▷ This includes the following questions:

  ▷ How does the quark population build up?

  ▷ How does it equilibrate? How fast?

  ▷ How are photons produced in a fast changing system?