HADRONIC MATTER AT HIGH BARYON DENSITY

J. Wambach
TU Darmstadt

QM-2002, July 24, 2002

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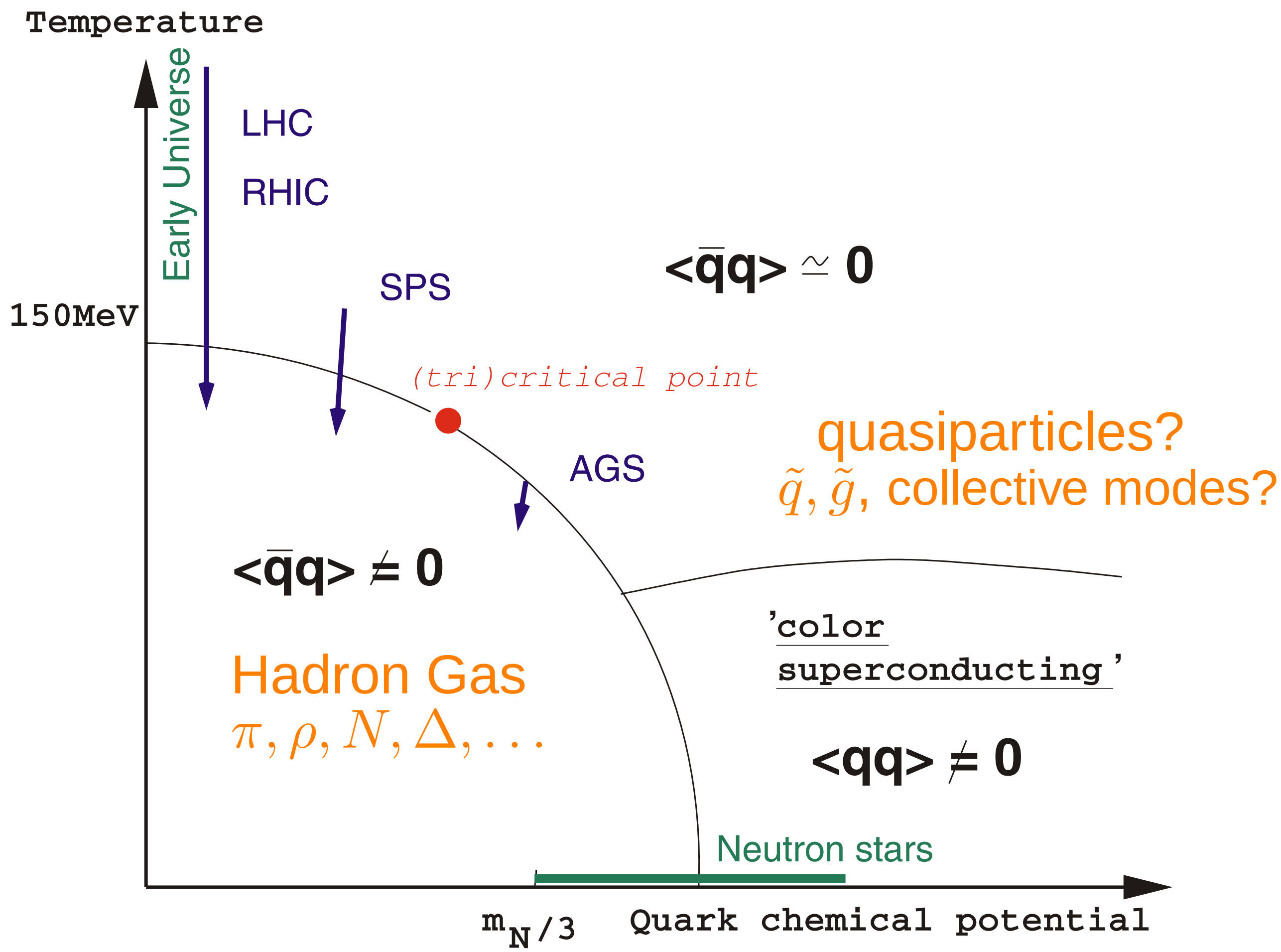
- Motivation
- Restoration of Chiral Symmetry
- Chiral Symmetry and Light Hadrons
- Fluctuations and Spectral Functions
- Experimental Signatures

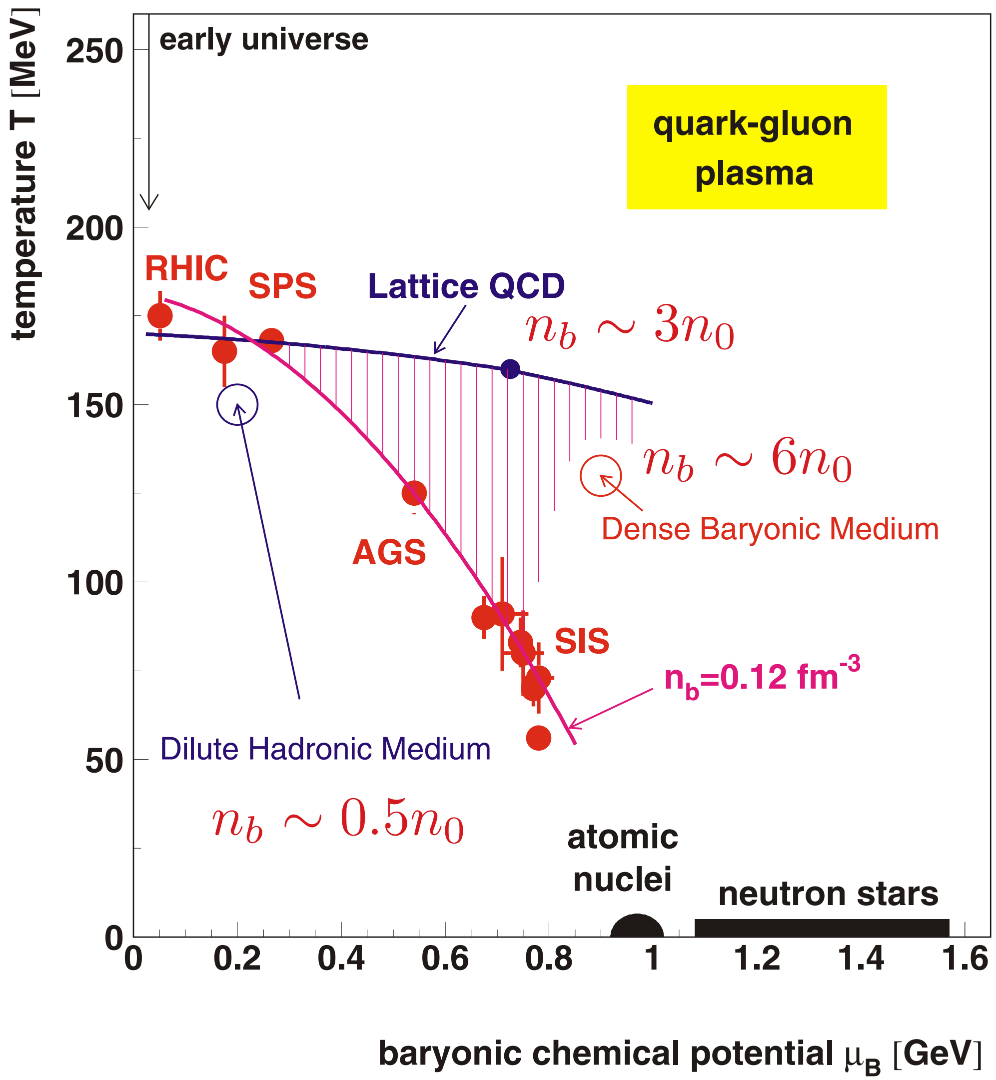
QCD Phase Diagram

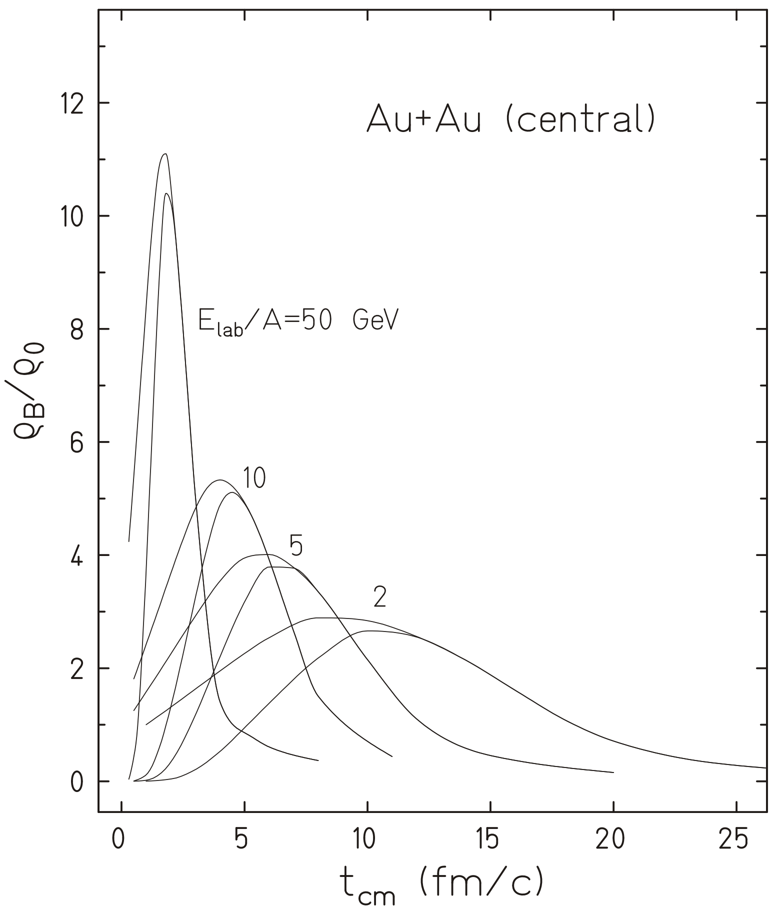


Issues

- Nature of the QCD phase transitions
  - where is the (Tri) critical point?
  - how strong is the 1.-order transition?
  - critical fluctuations?
  - Deconfinement=chiral restoration?
- Signatures
  - bulk thermodymanic variables
  - in-medium hadronic spectral functions
  - relation to deconfinement & chiral restoration?
  - relevance for neutron stars
- Theoretical tools
  - lattice QCD @ finite μ_q
  - effective field theories
  - non-perturbative many-body methods

QCD Phase Diagram





B. Friman et al. EJPA 3 (1998) 165

Chiral Symmetry Restoration

Hadron Gas

$$\frac{\langle\langle \bar{q}q \rangle\rangle}{\langle \bar{q}q \rangle} = 1 - \sum_h \frac{\Sigma_h \varrho_h^s(\mu, T)}{F_\pi^2 M_\pi^2}$$

$$\Sigma_h = m_q^\circ \frac{\partial M_h}{\partial m_q^\circ} = M_\pi^2 \frac{\partial M_h}{\partial M_\pi^2}; \qquad \varrho_h^s(\mu, T) = \frac{\partial \Omega(\mu, T)}{\partial M_h}$$

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ideal gas (pions, nucleons)

$$\frac{\langle\langle \bar{q}q \rangle\rangle}{\langle \bar{q}q \rangle} \approx 1 - \frac{T^2}{8F_\pi^2} - 0.3 \frac{n}{n_0} \dots$$

chiral pert. theory

Chiral Symmetry Restoration

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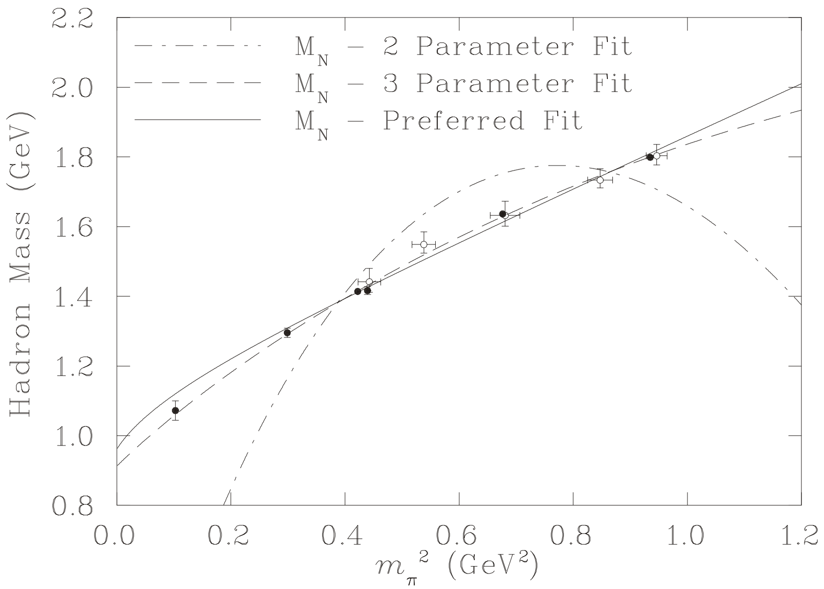
chiral pert. theory

but near the phase boundary

- proliferation of states!
- hadronic interactions!

Chiral Symmetry and Light Hadrons

nucleon mass on the lattice



D.B. Leinweber et al. PRD 61 (2000) 074502

$$M_N = aM_q + f_{na}(m_q^\circ)$$

$$\Sigma_N = m_q^\circ\left(a\frac{\partial M_q}{\partial m_q^\circ} + \frac{\partial f_{na}}{\partial m_q^\circ}\right)$$

Chiral Symmetry and Light Hadrons

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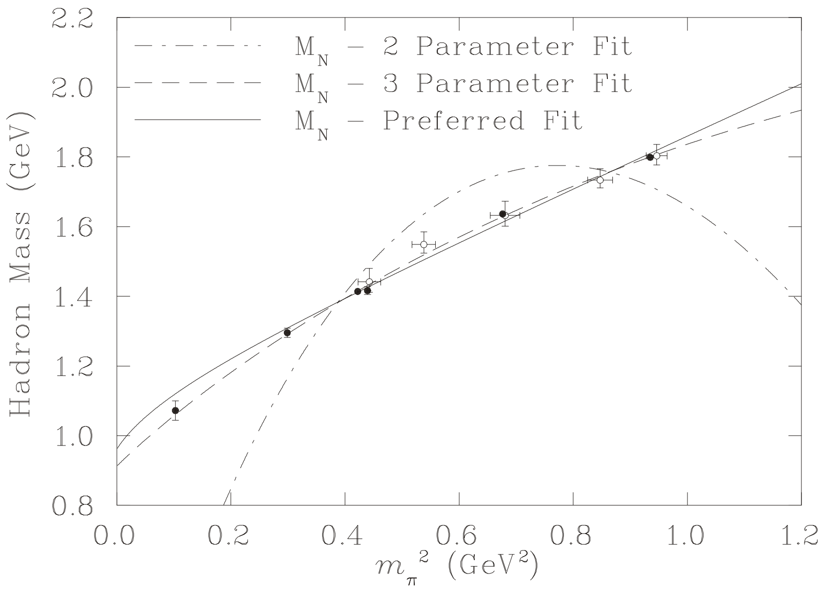

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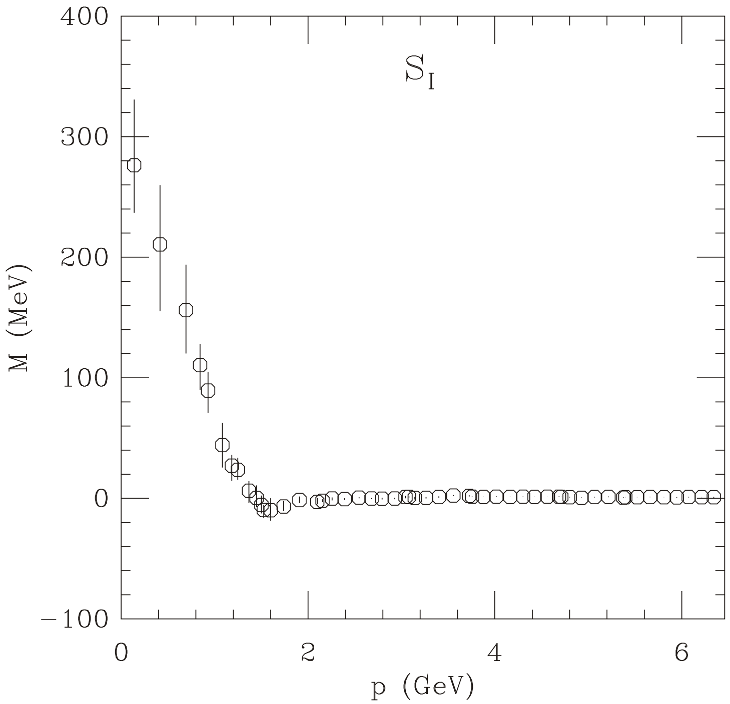
quark mass on the lattice



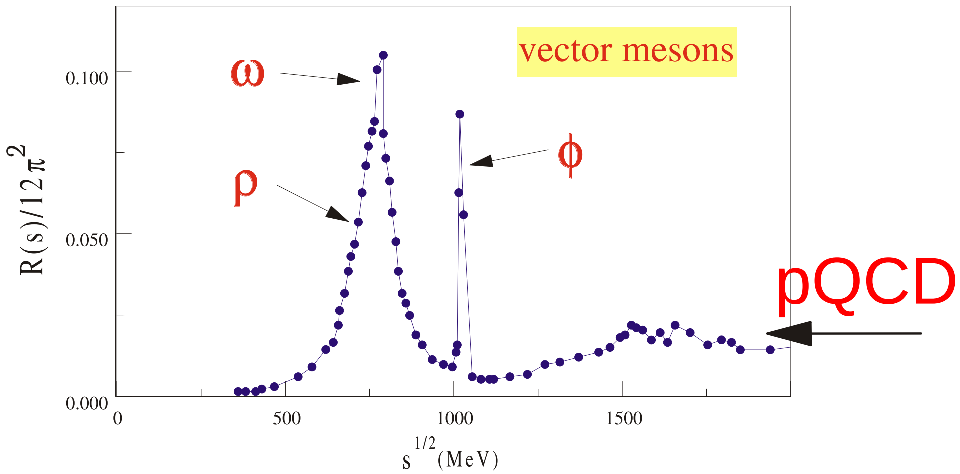
J. Skullerud et al. PRD 64 (2001) 074508

$$S_q^E(p) = \frac{Z_q(p^2)}{i\gamma_\mu p_\mu + M_q(p^2)}$$

vanishing of M_q at large p leads to parity doubling!

mesons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



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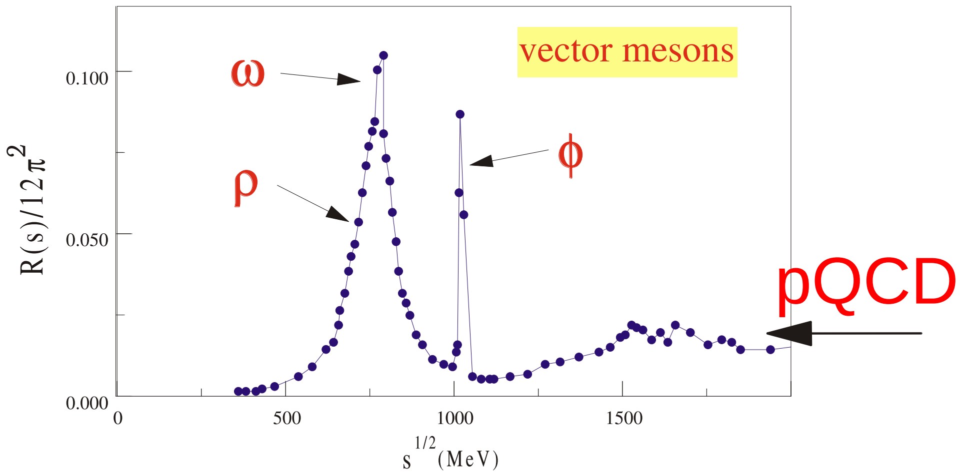
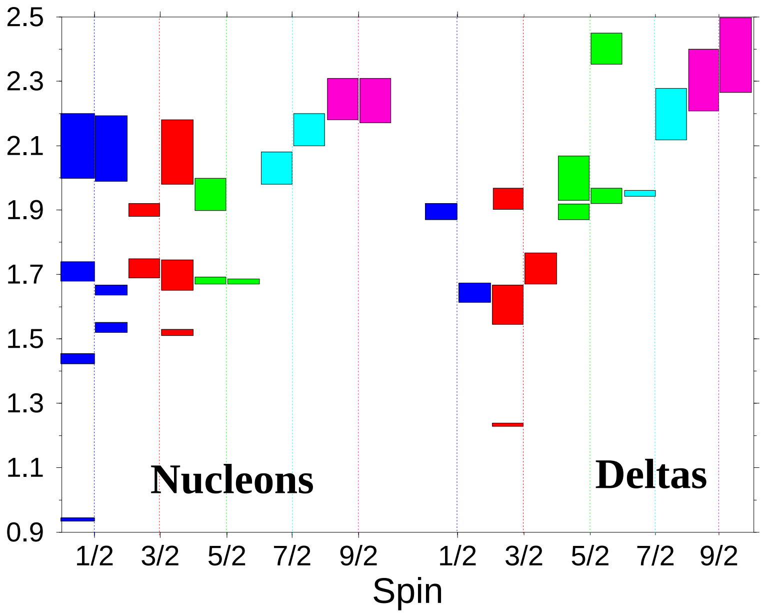


baryons



T.D. Cohen and L.Ya. Glozman, IJMP A16 (2001) 1327

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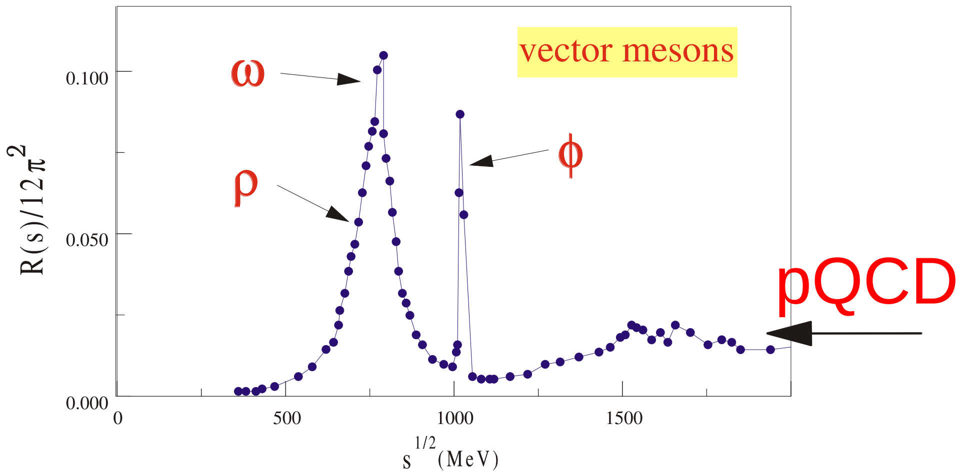

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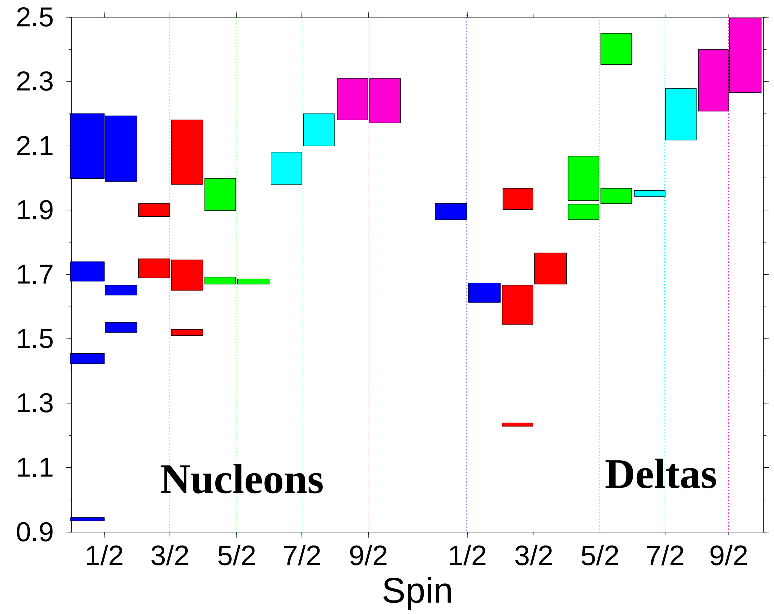

T.D. Cohen and L.Ya. Glozman, IJMP A16 (2001) 1327

- chiral breaking for $J^p = \frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$
- high-energy states decouple ($\Sigma_h \approx 0$)
- limited number of degrees of freedom!

Fluctuations

static susceptibilities reflect criticality

scalar:

$$\chi_S = \frac{V}{T}(\langle\!\langle(\bar{q}q)^2\rangle\!\rangle - \langle\!\langle\bar{q}q\rangle\!\rangle^2) = T\frac{\partial^2\Omega(\mu,T)}{\partial m_q^{\circ 2}}$$

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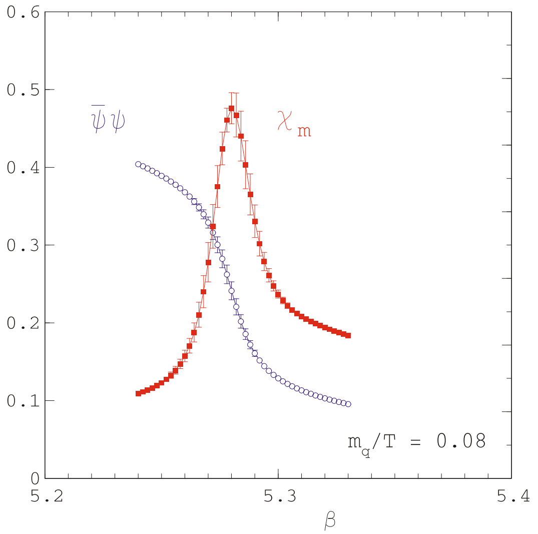

F. Karsch, hep-lat/0106019

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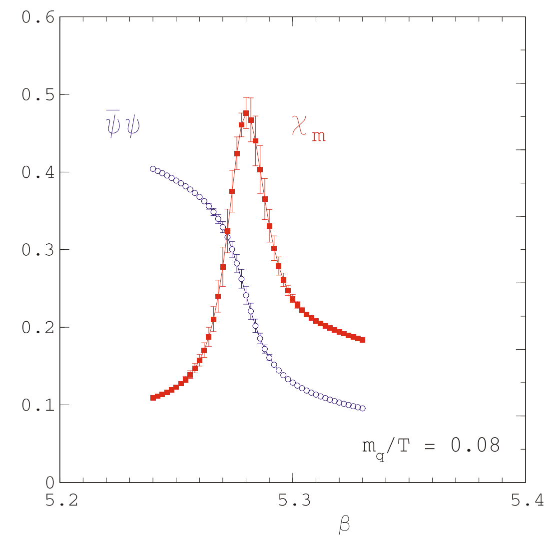

F. Karsch, hep-lat/0106019

vector/axialvector:

$$\chi_V = \frac{V}{T}(\langle\langle(\bar{q}\gamma^0\vec{\tau}q)^2\rangle\rangle - \langle\langle\bar{q}\gamma^0\vec{\tau}q\rangle\rangle^2) = T\frac{\partial^2\Omega(\mu,T)}{\partial\mu_V^2}$$

$$\chi_A = \frac{V}{T}(\langle\langle(\bar{q}\gamma^5\gamma^0\vec{\tau}q)^2\rangle\rangle - \langle\langle\bar{q}\gamma^5\gamma^0\vec{\tau}q\rangle\rangle^2) = T\frac{\partial^2\Omega(\mu,T)}{\partial\mu_A^2}$$

Fluctuations

static susceptibilities reflect criticality

scalar:

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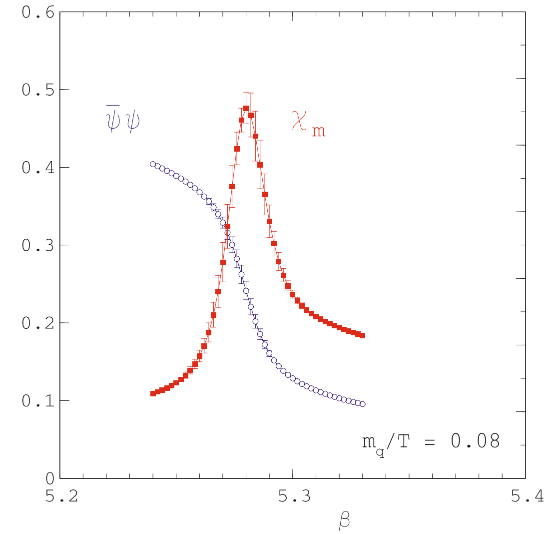

F. Karsch, hep-lat/0106019

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$$\chi_V = \frac{V}{T}(\langle\langle(\bar{q}\gamma^0\vec{\tau}q)^2\rangle\rangle - \langle\langle\bar{q}\gamma^0\vec{\tau}q\rangle\rangle^2) = T\frac{\partial^2\Omega(\mu,T)}{\partial\mu_V^2}$$

$$\chi_A = \frac{V}{T}(\langle\langle(\bar{q}\gamma^5\gamma^0\vec{\tau}q)^2\rangle\rangle - \langle\langle\bar{q}\gamma^5\gamma^0\vec{\tau}q\rangle\rangle^2) = T\frac{\partial^2\Omega(\mu,T)}{\partial\mu_A^2}$$

correlators:

$$\chi_i = VT\lim_{\omega,\vec{q}\to 0} D_i(\omega,\vec{q}); \quad D_i(\omega,\vec{q}) = i\int d^4x\, e^{iqx}\theta(x_0)\langle\langle[J_i(x),J_i(0)]\rangle\rangle$$

Fluctuations of the Chiral Condensate

hadronic spectral functions

$$\chi_S = VT \lim_{\omega,\vec{q}\to 0} \int_0^\infty d\omega \rho_S(\omega,\vec{q}); \quad \rho_S(\omega,\vec{q}) = -\frac{1}{\pi}\mathrm{Im}D_S(\omega,\vec{q})$$

$\longrightarrow$ soft mode spectroscopy!

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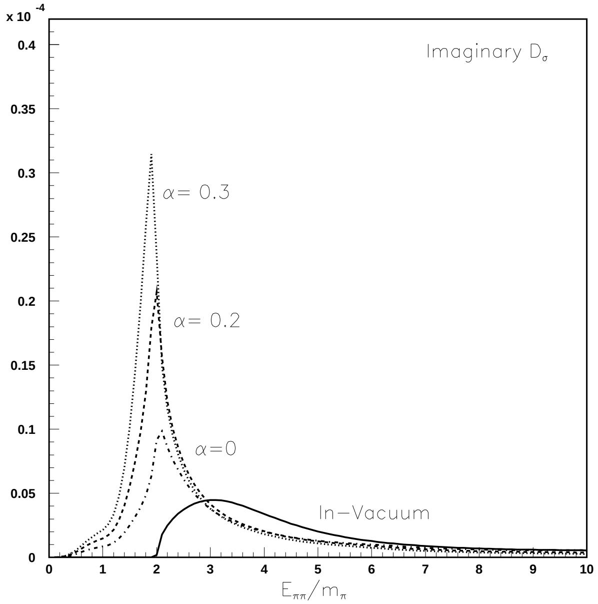
scalar spectral function



Z. Aouissat et al. PRC 61 (2000) 12202

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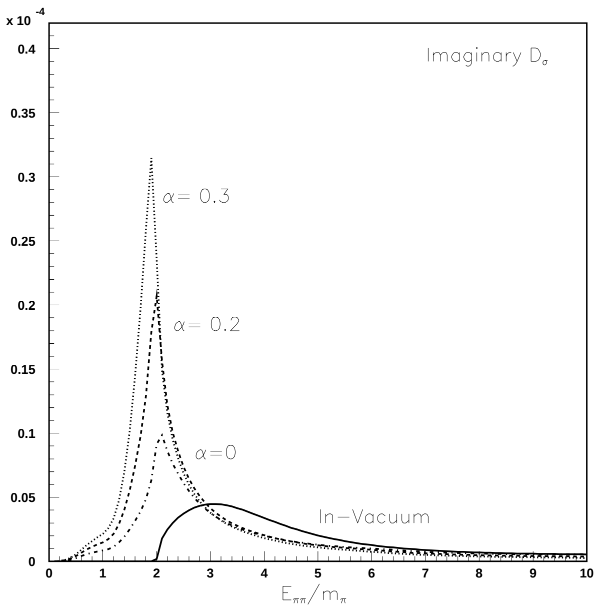
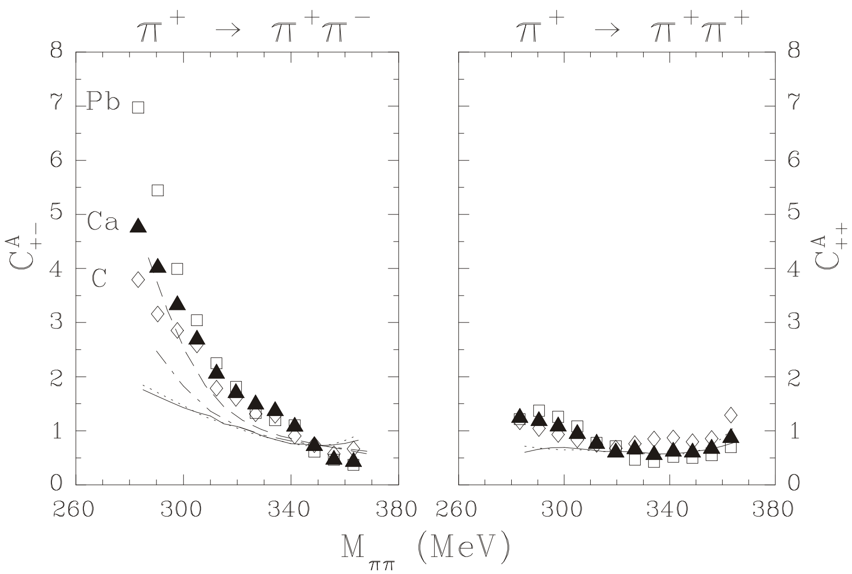


Z. Aouissat et al. PRC 61 (2000) 12202

$$C^A_{\pi\pi} = \frac{M^A_{\pi\pi}}{\sigma^A_T} \Big/ \frac{M^p_{\pi\pi}}{\sigma^p_T}$$



F. Bonutti et al. NPA 677 (2000) 213

Dilepton Rates

$$\frac{dN_{l^+l^-}}{d^4x d^4q} = L^{\mu\nu}(q)\left(\frac{-1}{\pi}\right)\mathrm{Im}D^{\mathrm{elm}}_{\mu\nu}(\omega,\vec{q})$$

$$D^{\mathrm{elm}}_{\mu\nu}(\omega,\vec{q}) = i\int d^4x\, e^{iqx}\theta(x_0)\langle\langle[J^{\mathrm{elm}}_{\mu}(x), J^{\mathrm{elm}}_{\nu}(0)]\rangle\rangle$$

$$J^{\mathrm{em}}_{\mu} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$$

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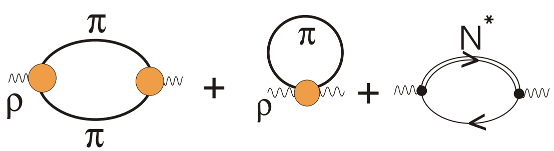
Vector Dominance

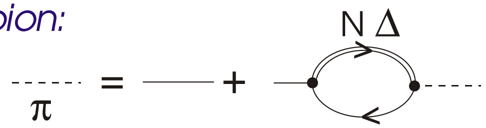
$$J^{\mathrm{v}}_\mu(x) = -\frac{M^2_\rho}{g}\rho_\mu(x)$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2}\mathcal{D}_\mu\vec{\pi}\cdot\mathcal{D}^\mu\vec{\pi} - \frac{1}{2}M^2_\pi\vec{\pi}\cdot\vec{\pi} - \frac{1}{8}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}M^2_\rho\rho_\mu\rho^\mu$$

rho meson:



pion:



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Vector Dominance

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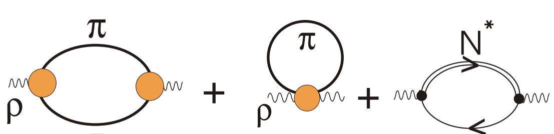

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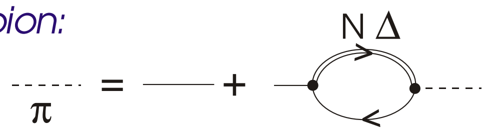
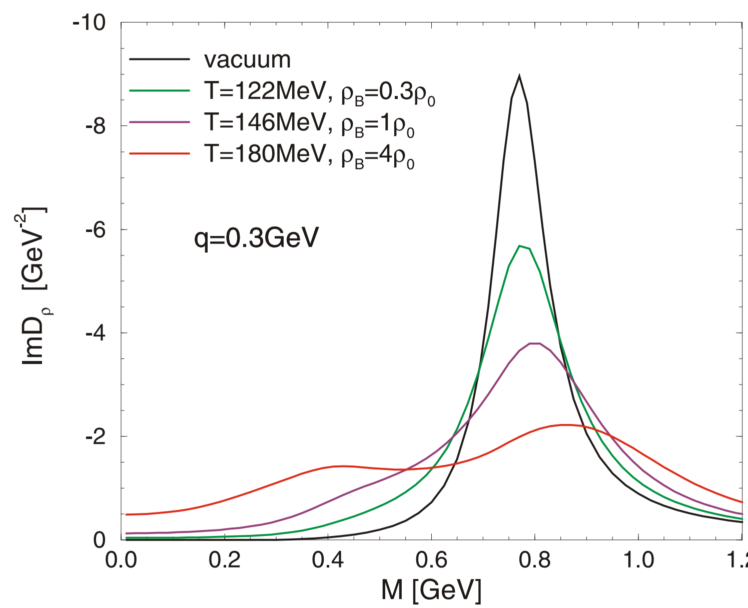
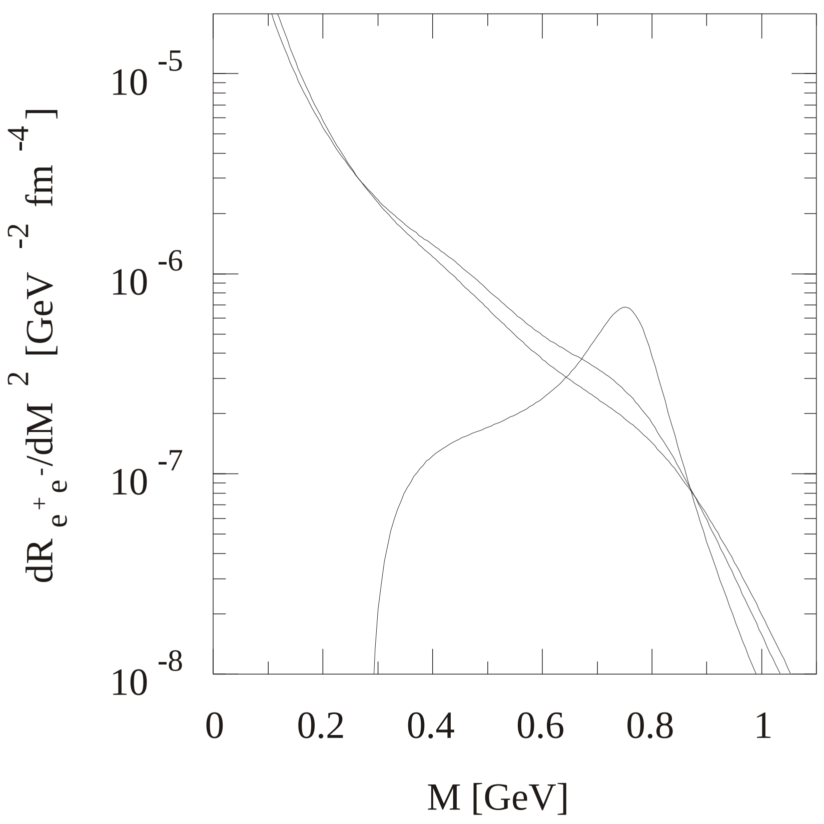


ρ-meson 'melts'



Dilepton Rates

close to rate of free quark gas



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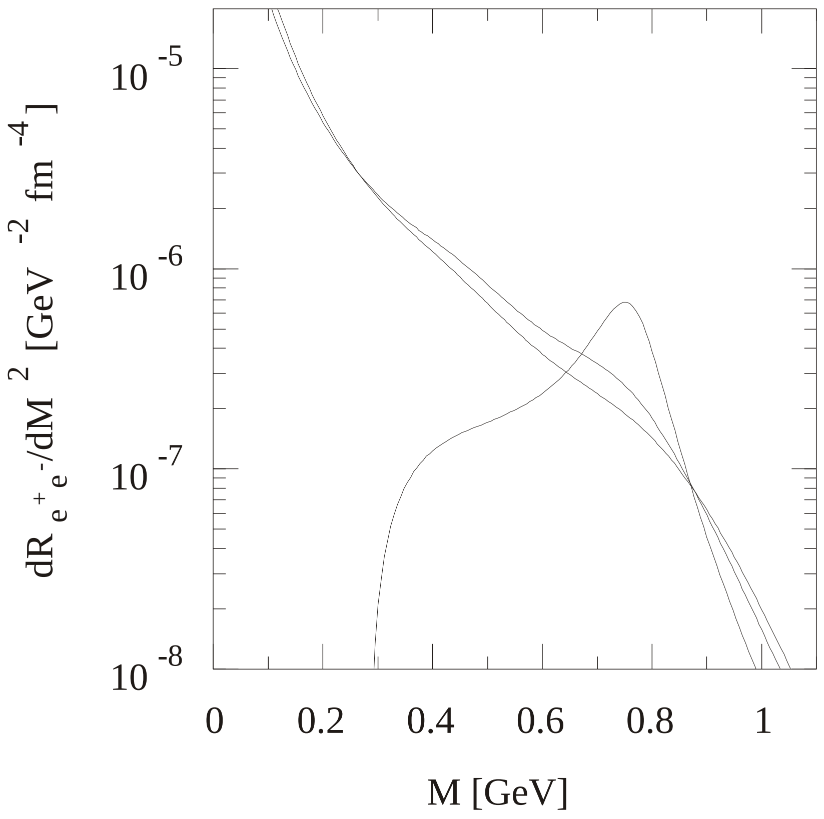
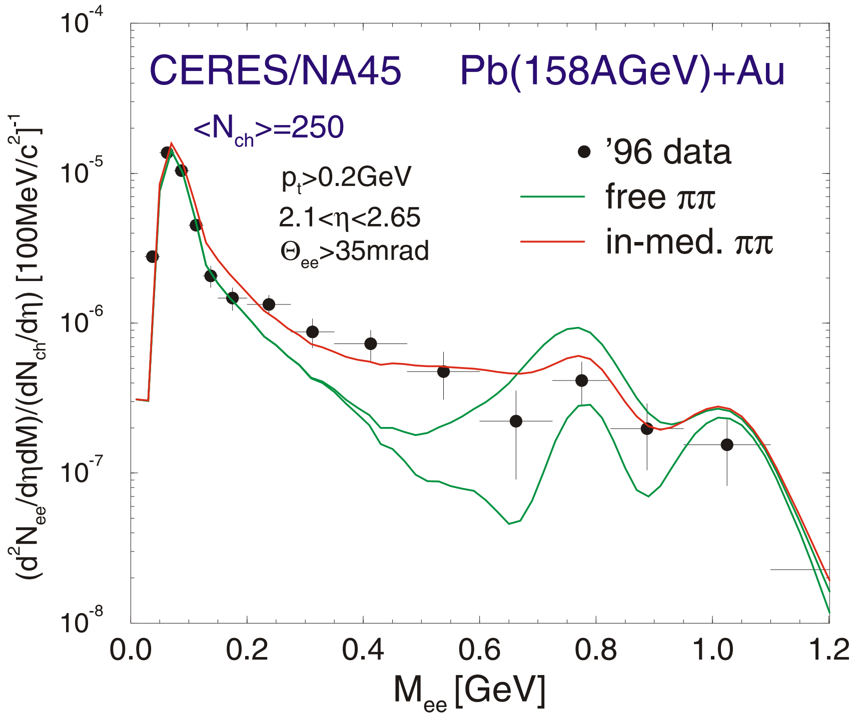


convolute over expanding fireball



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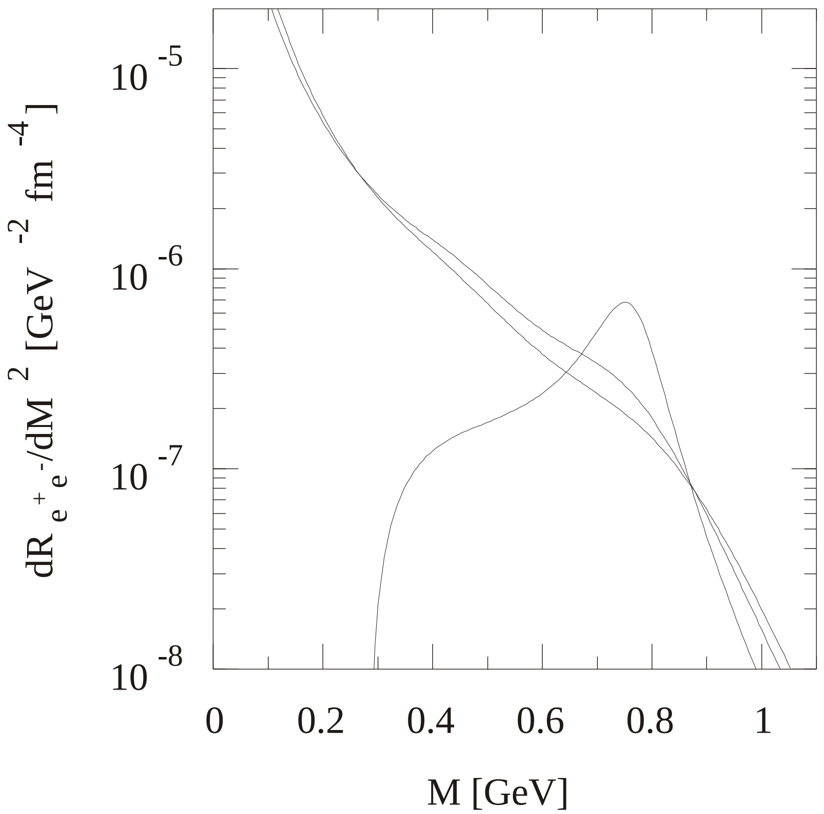

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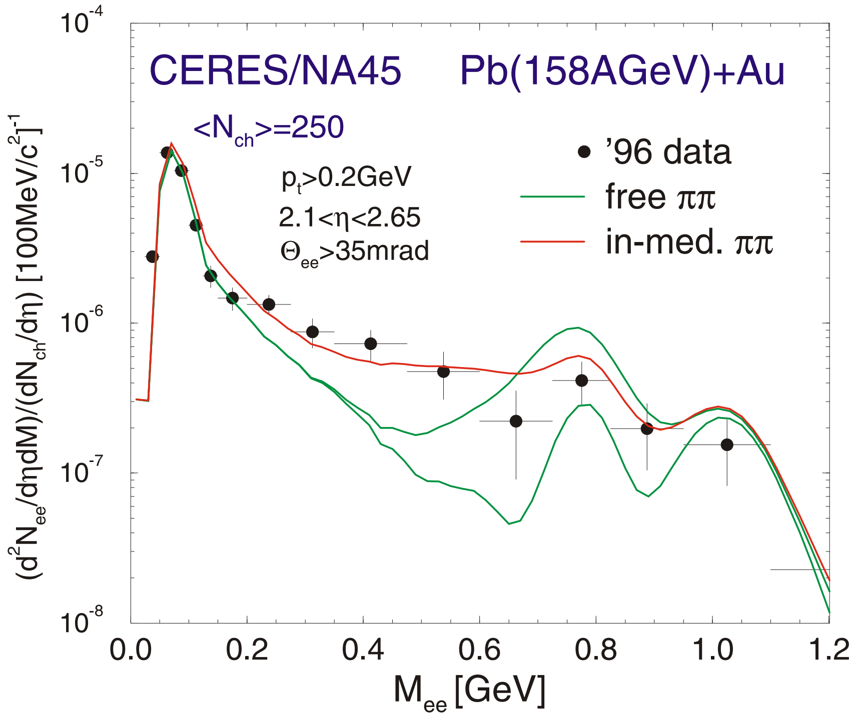
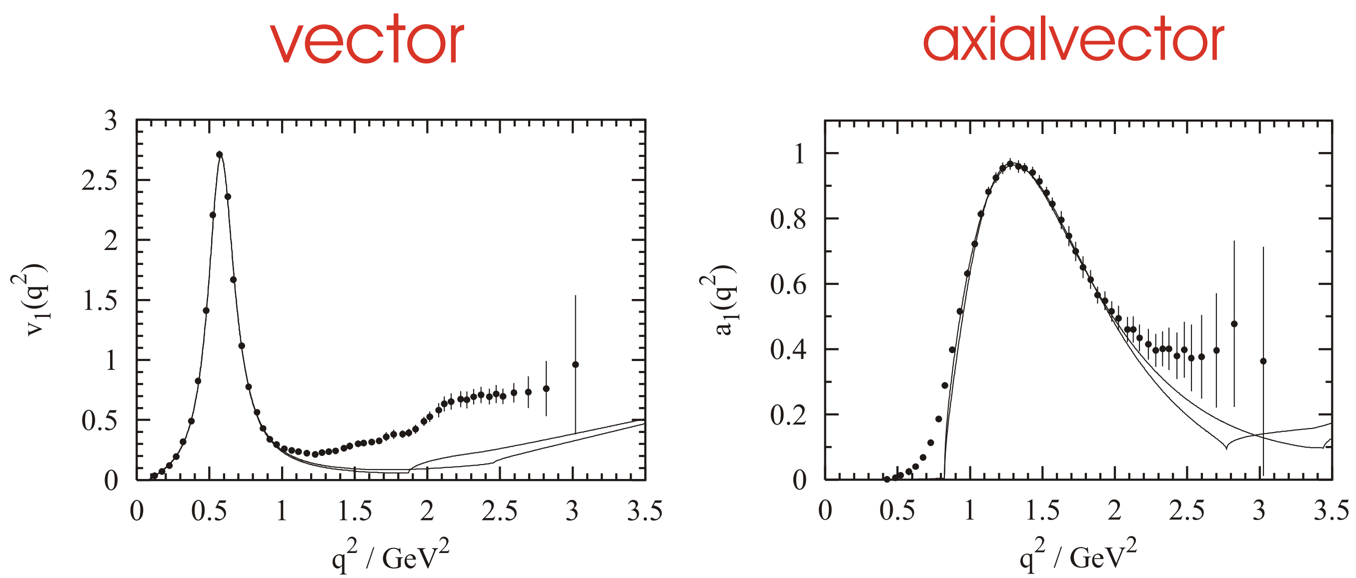


chiral symmetry restoration?

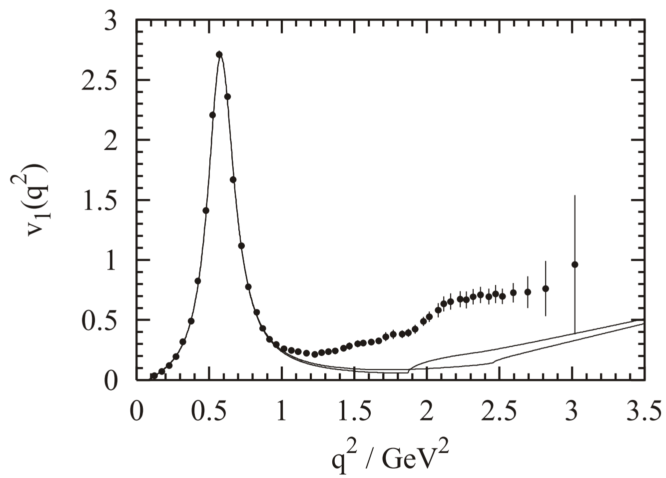
Vector Mesons and Chiral Symmetry



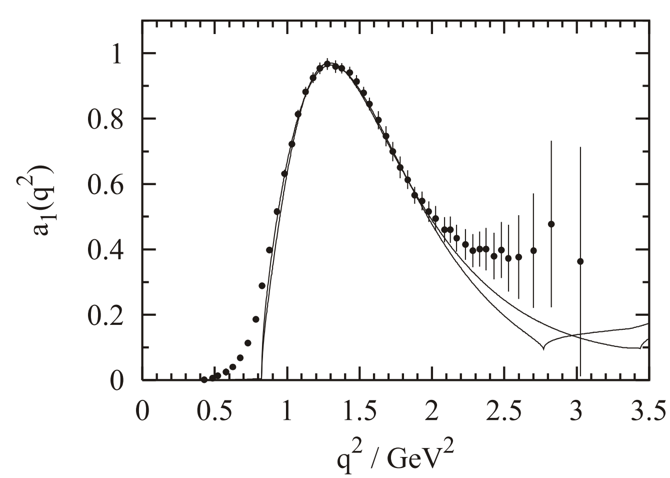
R. Barate et al (ALEPH) EJP C4 (1998) 409

Vector Mesons and Chiral Symmetry

vector



axialvector



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Weinberg sum rules

$$\int_0^\infty \frac{ds}{s}\left(\rho_V^\circ(s) - \rho_A^\circ(s)\right) = F_\pi^2; \quad \int_0^\infty ds\left(\rho_V^\circ(s) - \rho_A^\circ(s)\right) = 0$$

$$M_\rho^2 = a g_\rho^2 F_\pi^2; \quad a = \left(1 - \frac{M_\rho^2}{M_{a_1}^2}\right)^{-1}$$

$$a \sim 2.2$$

M. Urban et al NPA 697 (2001) 338

Vector Mesons and Chiral Symmetry

HLS model: M. Bando et al. PRL 54 (1985) 1215

$$\xi_{L,R} = e^{i\sigma/F_\sigma} e^{\mp i\pi/F_\pi}; \quad a \equiv F_\sigma^2/F_\pi^2$$

$$D_\mu \xi_\mathrm{L} = \partial_\mu \xi_\mathrm{L} - ig\rho_\mu \xi_\mathrm{L} + i\xi_\mathrm{L}\mathcal{L}_\mu \ ,$$
$$D_\mu \xi_\mathrm{R} = \partial_\mu \xi_\mathrm{R} - ig\rho_\mu \xi_\mathrm{R} + i\xi_\mathrm{R}\mathcal{R}_\mu$$

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$$D_\mu \xi_{\rm R} = \partial_\mu \xi_{\rm R} - i g \rho_\mu \xi_{\rm R} + i \xi_{\rm R} \mathcal{R}_\mu$$

$$\mathcal{L}_{\rm eff} = F_\pi^2 \, {\rm Tr} \left[\hat{\alpha}_{\perp\mu} \hat{\alpha}_\perp^\mu \right] + {\rm F}_\sigma^2 \, {\rm Tr} \left[\hat{\alpha}_{\parallel\mu} \hat{\alpha}_\parallel^\mu \right] + \mathcal{L}_{\rm kin}(\rho_\mu)$$
$$\hat{\alpha}^\mu_{\perp,\parallel} = (D_\mu \xi_{\rm R} \cdot \xi_{\rm R}^\dagger \mp D_\mu \xi_{\rm L} \cdot \xi_{\rm L}^\dagger)/(2i)$$

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- RG-flow of the scale Λ M. Harada et al. PRL 86 (2001) 757

$$F_\pi^2(\Lambda); \quad a(\Lambda); \quad g^2(\Lambda)$$

- matching of the vector- and axialvector correlators

$$\rho_{V,A}^{\rm HLS}(\Lambda_0^2) = \rho_{V,A}^{\rm QCD}(\Lambda_0^2); \qquad \Lambda_0 \sim 1 \text{ GeV}$$

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HLS model: M. Bando et al. PRL 54 (1985) 1215

$$\xi_{L,R} = e^{i\sigma/F_\sigma} e^{\mp i\pi/F_\pi}; \quad a \equiv F_\sigma^2/F_\pi^2$$

$$D_\mu \xi_{\mathrm{L}} = \partial_\mu \xi_{\mathrm{L}} - ig\rho_\mu \xi_{\mathrm{L}} + i\xi_{\mathrm{L}} \mathcal{L}_\mu \; ,$$
$$D_\mu \xi_{\mathrm{R}} = \partial_\mu \xi_{\mathrm{R}} - ig\rho_\mu \xi_{\mathrm{R}} + i\xi_{\mathrm{R}} \mathcal{R}_\mu$$

$$\mathcal{L}_{\mathrm{eff}} = F_\pi^2 \, \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu} \hat{\alpha}_\perp^\mu\right] + \mathrm{F}_\sigma^2 \, \mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu} \hat{\alpha}_\parallel^\mu\right] + \mathcal{L}_{\mathrm{kin}}(\rho_\mu)$$
$$\hat{\alpha}_{\perp,\parallel}^\mu = (D_\mu \xi_{\mathrm{R}} \cdot \xi_{\mathrm{R}}^\dagger \mp D_\mu \xi_{\mathrm{L}} \cdot \xi_{\mathrm{L}}^\dagger)/(2i)$$

- RG-flow of the scale Λ M. Harada et al. PRL 86 (2001) 757

$$F_\pi^2(\Lambda); \quad a(\Lambda); \quad g^2(\Lambda)$$

- matching of the vector- and axialvector correlators

$$\rho_{V,A}^{\mathrm{HLS}}(\Lambda_0^2) = \rho_{V,A}^{\mathrm{QCD}}(\Lambda_0^2); \qquad \Lambda_0 \sim 1 \text{ GeV}$$

- leads to 'BR scaling' @ finite T and μ

$$\frac{M_\rho^*}{M_\rho} = \frac{\langle\langle \bar{q}q \rangle\rangle}{\langle \bar{q}q \rangle}$$

Vector Mesons and Chiral Symmetry

'gauged' $O(4)$ model: $\phi = (\phi_0, \vec{\phi})$

$$\mathcal{M} = \phi_0 + i\vec{\tau}\vec{\phi} = Ae^{i\vec{\tau}\vec{\theta}}, \qquad \mathcal{D}^\mu = \partial^\mu - igY^\mu; \quad Y^\mu = \vec{\rho}^\mu\vec{\tau} + \vec{a}_1^\mu\vec{\tau}_5$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{4}\text{Tr}[\mathcal{D}_\mu\mathcal{M}\mathcal{D}^\mu\mathcal{M}^\dagger - \mu^2\mathcal{M}\mathcal{M}^\dagger + \cdots - \frac{1}{8}F_{\mu\nu}F^{\mu\nu} + \frac{M_0^2}{4}Y_\mu Y^\mu]$$

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tree level results:

$$M_\rho^2 = M_0^2 + h_2\langle\langle\phi_0\rangle\rangle^2; \quad M_{a_1}^2 = M_0^2 + (h_1 + h_2)\langle\langle\phi_0\rangle\rangle^2$$

$\Longrightarrow$ $\delta M_i \sim \mathcal{O}(T^2)$ for $M_0 = 0$

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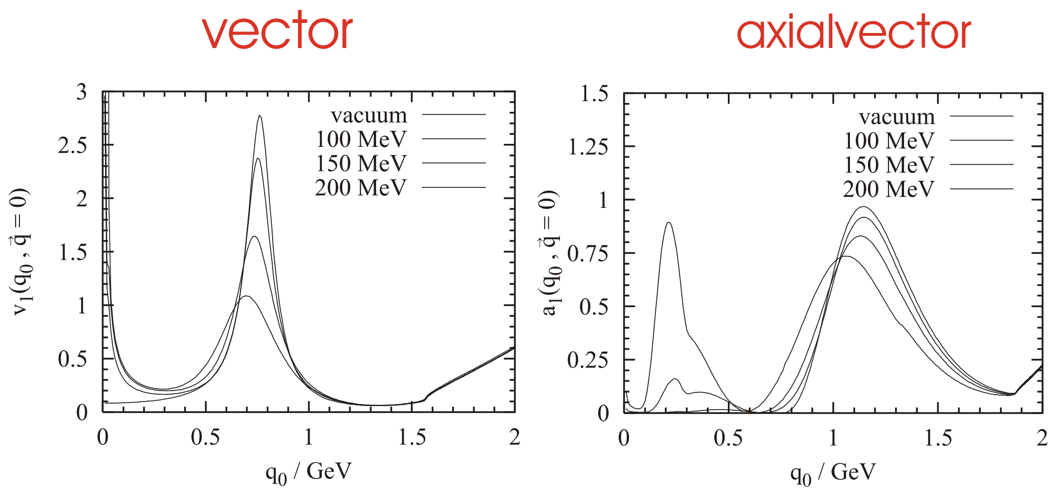
one-loop results:

- mixing theorem statisfied
- $\delta M_i \sim \mathcal{O}(T^4)$



M. Urban et al PRL 88 (2002) 042002